Physics 17 Part A

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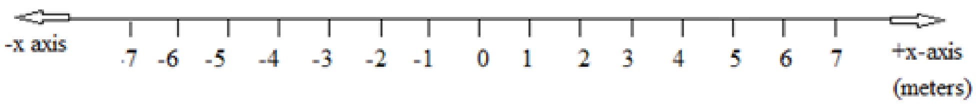
Horizontal Motion



Consider an object moving along the horizontal line above. This line is called “the x-axis.” Various points along that line mark distances to the right and left from the “0” reference zero mark, and are called “x-coordinates.” The “units” of these coordinates are “meters” (m). The meter unit is the Standard International (SI) Unit for length and distance, and is approximately 3.28 feet.

Objects traveling along this axis are initially at $x = 0$. At the end of the traveling period the object is (usually) at some other x-coordinate, x.

Travel times are symbolized as t, whose SI units are “seconds” (s)

During the period of time the object is traveling, its “average velocity” is defined and symbolized as shown below:

Average Velocity:

$\bar{v} = x/t$ (v-bar)

$x = \bar{v}\, t$

<u>Example</u>:

At the end of a five-second period of travel an object is at x = 30 m. What was its average velocity?

$$\bar{v} = x/t$$
$$= 30 \text{ m}/5 \text{ s}$$
$$= 6 \text{ m/s}$$

Note: the SI units of average velocity are meters per second (m/s).

Example A: An object moves to the left, arriving at x = -180 m at the end of a six-second travel time. What was its average velocity? $\bar{v}$ = x /t = -180 m /6 s = -30 m/s The negative velocity indicates the object was moving to the left, in the negative-x direction during that time.
Example B: An object moving to the right travels 36 meters in 12 seconds. What was the object's average velocity? x = 36 m $\bar{v}$ = x /t = 36 m/ 12 s = 3 m/s The positive velocity indicates the object was moving to the right during that time.

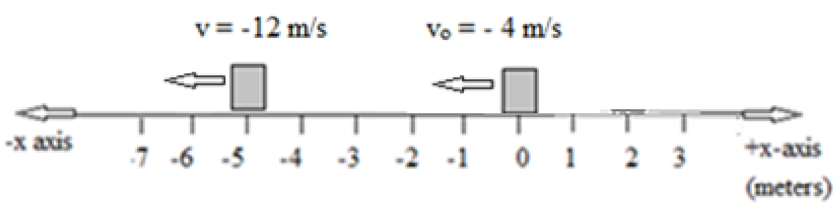
Instantaneous Velocity

The "instantaneous" velocity of an automobile moving along a road is the reading of its speedometer at some instant, prefixed by a negative sign if the car is moving to the left; otherwise, if it's moving to the right, its velocity is positive. From now on, when we refer to instantaneous velocity, we will call it by a shorter name–"velocity."

We symbolize an object's initial and final velocities as shown below:

Initial Velocity: v_o
Final Velocity: v

The zero subscript on the initial velocity reminds us that the initial location of the object is the zero-reference mark; it also reminds us that when the travel time is specified, or measured, the stopwatch is set to zero at the beginning of the motion. An example of a moving object's initial and final velocities is found in the figure below:



Middle-Value Rule

We saw earlier how to calculate the average velocity of an object using the equation below:

$$\bar{v} = x/t$$

There is another way to calculate the average velocity.

The average velocity is found using the "middle-value" rule:

$$\bar{v} = \tfrac{1}{2}(v_o + v)$$

This equation gives a number that is mid-way between the initial and final velocities.

Example A:	Example B:
An object's velocity changes from 20 m/s to 80 m/s. What was its average velocity? $\bar{v} = ½ (20 + 80)$ $= 50$ m/s Note that 50 is "midway" between 20 and 80: It's 30 above 20, and 30 below 80.	The average velocity of an object is 24 m/s; its final velocity is 10 m/s. What was its initial velocity? $\bar{v} = ½ (v_o + v)$ $24 = ½ (v_o + 10)$ $v_o = 38$ m/s

Acceleration

When an object is accelerating, its velocity is changing–either increasing, or decreasing. The average acceleration is the ratio of the change in velocity, divided by the time required for the change to occur:

$$\bar{a} = (v - v_o)/t$$

The SI units of acceleration are meters per second, per second, or m/s^2.

For the remainder of this course, our accelerations will not be changing, so it will make no sense to talk about the average value of a quantifiable property whose value never changes. Therefore, we can remove the "bar" from above the "a":

$$a = (v – v_o)/ t$$

The velocity equation below is the same as the one above, re-arranged.

$$v = v_o + at$$

Example A:	Example B:
An object whose initial velocity was 4.0 m/s accelerated at 2.0 m/s^2 for three seconds. What was its final velocity?	An object whose initial velocity was 4.0 m/s accelerated at -2.0 m/s^2 for five seconds. What was its final velocity?
$v = 4.0\ m/s + (2.0\ m/s^2)\ (3.0\ s)$ $= 4.0\ m/s + 6.0\ m/s$ $= 10.0\ m/s$	$v = 4.0\ m/s + (-2.0\ m/s^2)\ (5.0\ s)$ $= 4.0\ m/s - 10.0\ m/s$ $= -6.0\ m/s$

The tables below illustrate accelerations that are positive, negative, and zero.

Table 1

t (s)	v (m/s)
0	6
1	9
2	12
3	15
4	18
5	21
6	24
7	27

$a = 3\ m/s^2$

Table 2

t (s)	v (m/s)
0	15
1	12
2	9
3	6
4	3
5	0
6	-3
7	-6

$a = -3\ m/s^2$

Table 3

t (s)	v (m/s)
0	15
1	15
2	15
3	15
4	15
5	15
6	15
7	15

$a = 0\ m/s^2$

In Table 2, note that until time t = 5 seconds is reached, the object's velocity is positive, indicating that it's moving to the right. After 5 seconds, the velocity is negative, meaning that the object reversed direction and now is moving to the left.

<u>Example A</u>: An object's initial velocity is 30 m/s, and is accelerating at 2 m/s^2. After how many seconds will the velocity be 60 m/s? $v = v_o + at$ $60\text{ m/s} = 30\text{ m/s} + (2\text{ m/s}^2)\, t$ $t = 15\text{ s}$	<u>Example B</u>: An object's velocity is initially 20 m/s. It then begins accelerating at the rate of 5 m/s^2. (a) What is its velocity nine seconds later? $v = v_o + at$ $= 25\text{ m/s} + (5\text{ m/s}^2)\, 9\text{ s}$ $= 70\text{ m/s}$ (b) What is the object's x-coordinate at the end of that nine-second trip? $x = \bar{v}\, t$ $= ½\,(20 + 70)\, 9$ $= 405\text{ m}$

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<u>Example A</u>: An object initially at $x_o = 20$ m has a velocity $v_o = 6$ m/s. Three seconds later its velocity is $v = 18$ m/s. How far did the object travel in this time? $\bar{v} = ½\,(v_o + v)$ $= ½\,(6 + 18)$ $= 12\text{ m/s}$ $x = \bar{v}\, t$ $= (12\text{ m/s})\,(3\text{ s})$ $= 36\text{ m}$ The object didn't reverse direction, so the distance traveled equals the final x-coordinate: Distance = 36 m =	<u>Example B:</u> An object initially traveling at a velocity of 40 m/s begins to slow down at the rate - 4 m/s^2. How far will the object have traveled when it comes to rest? At rest, the object's velocity is zero: $v = v_o + at$ $0 = 40 - 4\, t$ $t = 10\text{ s}$ $\bar{v} = ½\,(v_o + v)$ $= ½\,(40 + 0)$ $= 20\text{ m/s}$ $x = \bar{v}\, t$ $= (20\text{ m/s})\,(10\text{ s})$ $= 200\text{ m}$

Below is derived another motion equation.

First, recall the two equations below:

$x = \frac{1}{2}(v_o + v)\,t$ (1)

$v = v_o + at$ (2)

Use (2) to replace v in (1):

$$\begin{aligned} x &= \tfrac{1}{2}\,[v_o + (v_o + at)\,]\,t \\ &= \tfrac{1}{2}\,(2v_o + at)\,t \\ &= v_o t + \tfrac{1}{2}\,at^2 \end{aligned}$$

$$x = v_o\,t + \tfrac{1}{2}\,at^2$$

<u>Example</u>:

An object initially is moving at 6 m/s and accelerating at 3 m/s^2. What is the x-coordinate of the object 8 seconds later?

$$\begin{aligned} x &= v_o t + \tfrac{1}{2}\,at^2 \\ &= 6\,(8) + \tfrac{1}{2}\,(3)\,8^2 \\ &= 144 \text{ m} \end{aligned}$$

Example:

An object is initially moving at 6 m/s velocity. It then begins accelerating at the rate of 4.0 m/s^2.

After how many seconds will the object be at x = 160 m?

$x = v_o t + \frac{1}{2} a t^2$
$160 = 6\,t + \frac{1}{2}(4)\,t^2$

$2\,t^2 + 6\,t - 160$

This is a quadratic equation, so it has two solutions:

Using a calculator equation solver, we get
t = 7.57 s and t = -10.57 s.

We ignore the negative solution as non-physical: there is no such thing as a negative event time.

One More Equation

Recall the equations below:
$v = v_o + at$
$x = ½ (v + v_o) t$

Solve the first equation for t, then substitute into the second equation to get

$$v^2 = v_o^2 + 2 ax$$

Example: An object initially moving at 12 m/s has a velocity of 34 m/s after traveling 900 m. What was its acceleration?

$34^2 = 12^2 + 2a (900)$
$a = 1.12 \text{ m/s}^2$

Horizontal Motion Equation Summary

$\bar{v} = ½ (v_o + v)$
$x = ½ (v_o + v) t$
$v = v_o + at$
$x = v_o t + ½ at^2$
$v^2 = v_o^2 + 2 ax$