Physics 17 Part B Dr. Joseph F. Alward

Horizontal Motion (Continued)

In the previous work involving only one object, we defined the object's initial x-coordinate to be zero. However, in the example below involving two objects initially separated from each other, it is not possible to have each object to initially be at the same location ($x = 0$), so we must use the following equation to allow for objects to have different x-coordinates:

$$x = x_o + v_o t + \frac{1}{2} a t^2$$

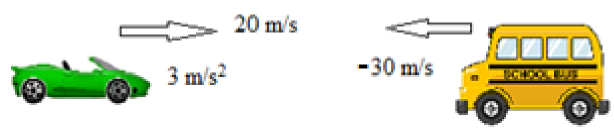
<u>Example</u>:

A car is traveling the wrong way on a straight, one-way street at a velocity of 20 m/s and accelerating at 3 m/s^2. We assume the direction of the car's travel is to the right, in the positive direction.

A bus located 500 m away on the same street is moving at a steady velocity of -30 m/s toward the car--in the negative x-direction.

After how many seconds will the two vehicles collide head-on?

We are free to let the initial x-coordinate of the car be $x_o = 0$. With this choice, however, the initial x-coordinate of the bus is $x_o = 500$ m. (We could have let $x_o = 0$ for the bus, and $x_o = -500$ m for the car; the results would be the same.)



$$x = x_o + v_o t + \frac{1}{2} a t^2 = 0 + 20\,t + \frac{1}{2}(3)\,t^2$$

$$x = x_o + v_o t + \frac{1}{2} a t^2 = 500 - 30t + 0$$

When the vehicles collide, their x-coordinates are the same:

$$20t + \frac{1}{2}(3)\,t^2 = 500 - 30t$$

$$t = 8.05 \text{ s}$$

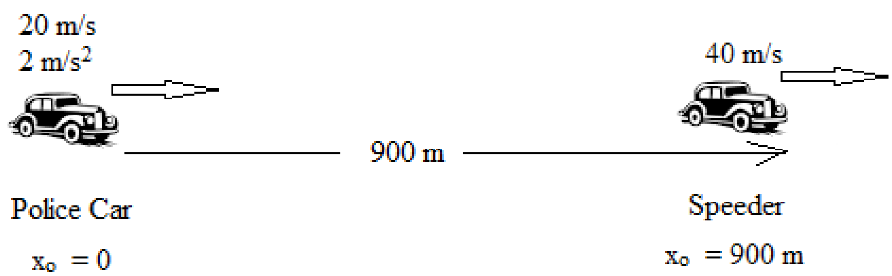
This quadratic equation has two solutions; the other one is -41.4 s. We reject this answer in favor of 8.05 s because negative event times are physically meaningless.

Example:

A police car moving at $v_o = 20$ m/s and accelerating at 2 m/s^2 is chasing a speeding automobile traveling at $v_o = 40$ m/s, and not accelerating. The distance between the two vehicles initially is 900 meters.

How much time will it take the police car to catch the speeder?

We choose $x_o = 0$ for the police car, and $x_o = 900$ m for the automobile.



Police car:

$x = x_o + v_o t + ½ at^2$
$= 0 + 20 t + ½ (2) t^2$
$= 20 t + t^2$

Speeder:

$x = x_o + v_o t + ½ at^2$
$= 900 + 40 t + 0$
$= 900 + 40 t$

When the speeder is caught, the x-coordinates of the two vehicles will be the same:

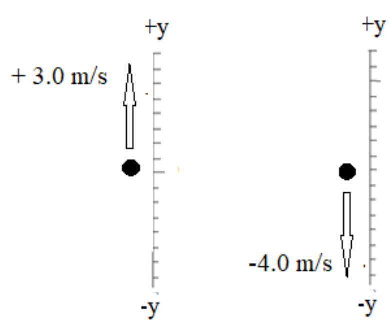
$20 t + t^2 = 900 + 40 t$
$t = -21.62$ s and 41.62 s

We reject the physically meaningless negative time.

Vertical Motion

In this section we will study the motion of a single object moving straight upward, or downward, under the influence only of Earth's gravitational pull: the object's motion will not be influenced by any propulsion device attached to it.

The equations for vertical motion are similar to the ones for horizontal motion, except we will label the line along which the objects travel upward or downward the "y-axis."



Objects moving upward have positive velocities, while objects moving downward have negative velocities.

The zero reference location is wherever the object was when it was first observed.

In all of the examples of vertical motion, the object will be moving upward--or downward--"freely," i.e., only under the influence of Earth's gravitational pull.

Objects rising or falling are accelerating at a rate called "the acceleration due to gravity." *The acceleration is negative not only for objects moving downward, but upward as well.*

$a = -9.8\ m/s^2$

The number 9.8 m/s^2 occurs so often in problems involving vertical motion that to save time we let the symbol "g" represent it. 9.8 m/s^2.

Horizontal	Vertical
$\overline{v} = ½ (v_o + v)$	$\overline{v} = ½ (v_o + v)$
$x = ½ (v_o + v) t$	$y = ½ (v_o + v) t$
$v = v_o + at$	$v = v_o - gt$
$v^2 = v_o^2 + 2 ax$	$v^2 = v_o^2 – 2gy$
$x = v_ot + ½ at^2$	$y = v_ot - ½ gt^2$

The vertical motion equations are the same as the horizontal motion equations, with a replaced by -g, and x replaced by y:

Example :

An object is thrown downward with an initial velocity v_o = -16 m/s from the top of a building 30 meters high. How long will it take to strike the ground?

The object is initially at the top of the building, so that is where the zero reference point is located. Consequently, the later y-coordinate of the object on the ground 30 meters below is *negative*: y = -30 m

$$y = v_o t - ½ gt^2$$
$$-30 = -16t - ½ (9.8)t^2$$
$$t = 1.33 \text{ s}$$

Example A:

An object is thrown upward with an initial velocity $v_o = 24$ m/s from the top of a building 70 meters high. How long will it take to strike the ground?

$y = v_o t - ½ gt^2$
$-70 = 24\ t - ½ (9.8)t^2$
This is a quadratic equation, so it has two solutions:

$t = 6.95$ s and $t = -2.05$ s

We reject the physically meaningless negative time solution.

Example B:	(c) After how many seconds will the bullet reach its maximum height?
A bullet is fired upward with an initial velocity of 60 m/s. (a) What will be its velocity three seconds later? $v = v_o - gt$ $= 60 - 9.8\ (3)$ $= 30.6$ m/s (b) How high did the bullet travel in those three seconds? $y = ½\ (v_o + v)\ t$ $= ½\ (60 + 30.6)\ 3$ $= 135.9$ m Another way: $y = v_o t - ½ gt^2$ $= 60(3) - ½\ (9.8)3^2$ $= 135.9$ m	When maximum height is reached, the object's velocity is (just for an instant) equal to zero. $v = v_o - gt$ $0 = 60 - 9.8\ t$ $t = 6.12$ s (d) What maximum height was reached by the bullet? $y = ½\ (v_o + v)\ t$ $y = ½\ (60 + 0)\ 6.12$ $= 184$ m Another way: $v^2 = v_o^2 – 2\ gy$ $0^2 = 60^2 – 2\ (9.8)\ y$ $y = 184$ m

Projectile Motion

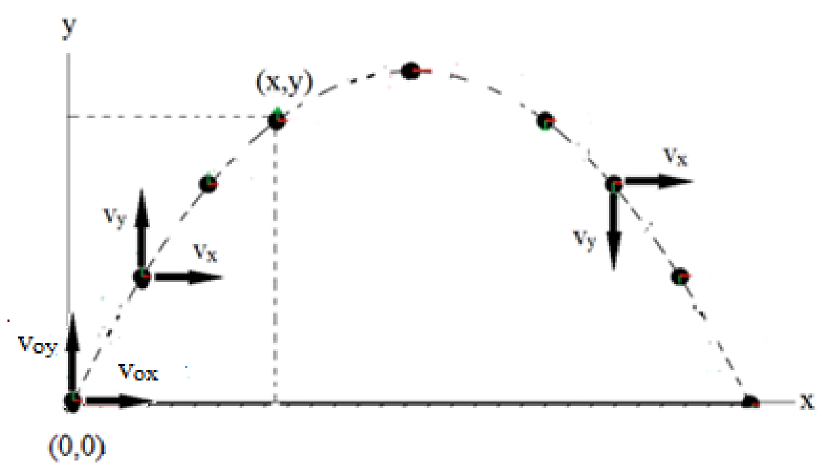
"Projectiles" are objects that are struck, thrown, launched, or fired forward and upward or downward, traveling only under the influence of gravity. Struck golf balls, shot arrows, and fired bullets execute projectile motion. A rocket, for example, does not execute projectile motion until all of its fuel has been exhausted and it's moving on its own--"freely"-- only under the influence of Earth's pull (gravity), without any propulsion device (rocket engine) aiding its motion.

In projectile motion, the object is not only moving horizontally, but also vertically. Therefore, there are two velocities-- horizontal, and vertical, as well as two kinds of directions of travel-- horizontal, and vertical. The path the projectile travels is called its "trajectory."

In all of the projectile motion problems we will study below, it is assumed that the initial x- and y-coordinates of the projectile are zero, i.e., the projectile is initially at the coordinates (0, 0).

In problems involving simultaneous horizontal and vertical movements, we will need to be able to distinguish between x- and y-motion variables. To this end, we use subscripts "x" and "y" on the variables, as shown in the tables below.

x	x-coordinate
y	y-coordinate
v_{ox}	Initial horizontal velocity
v_{oy}	Initial vertical velocity
v_y	Later vertical velocity

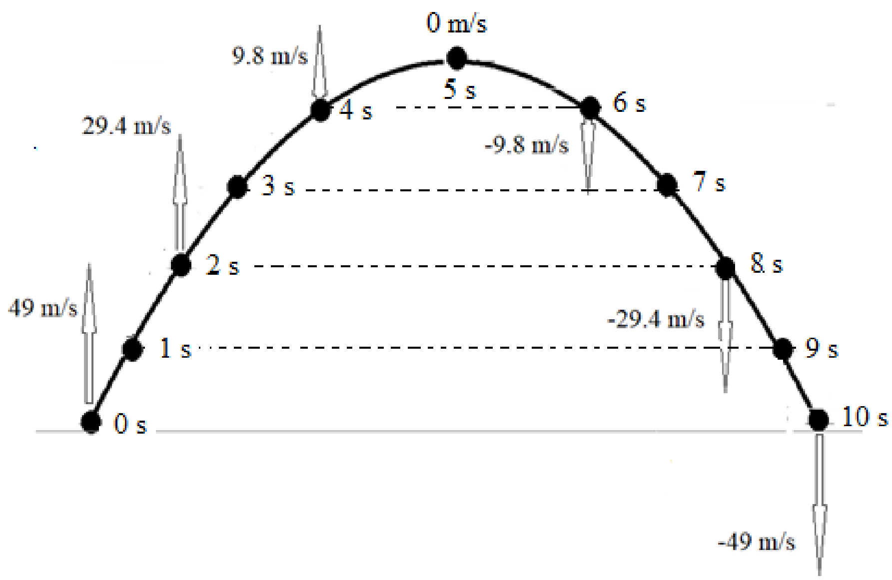


Vertical Component of Projectile Motion

The figure below shows how the vertical velocity changes in the case where the initial vertical velocity is 49 m/s. With each passing second on the way up, and then on the way down, the object's velocity is decreased by 9.8 m/s.

$$v_y = v_{oy} - gt$$

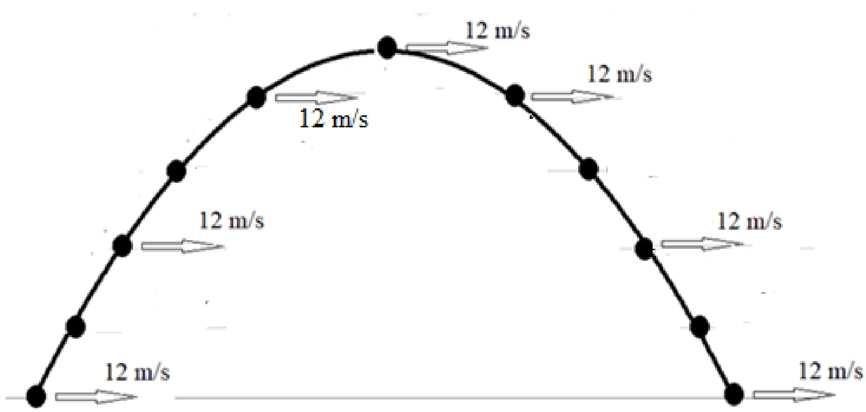
$$v_y = 49 \text{ m/s} - (9.8 \text{ m/s}^2)\, t$$



Horizontal Velocity in Projectile Motion

Unlike the vertical velocities that constantly are changing during projectile motion, *horizontal velocities in projectile motion never change,* assuming we can ignore the retardation effects of air resistance. In the example trajectory shown below, the ball's horizontal velocity throughout its motion is the same as it was initially:

$$v_x = v_{ox}$$
$$= 12 \text{ m/s}$$



Unless otherwise specified, all of the projectile motion problems will deal with projectiles fired from the ground (y = 0), over level ground, and landing on the ground (y = 0).

Total Flight Time

Calculate the total flight time for projectile motion of a golf ball traveling ground-to-ground over level ground.

v_{oy} = Initial vertical velocity

The golf ball begins its upward journey on the ground, at y = 0, and ends its journey on the ground, at y = 0.

$y = v_{oy}\, t - \frac{1}{2} gt^2$
$0 = v_{oy}\, t - \frac{1}{2} gt^2$ (t = total time)

Solve for t:
$t = 2\,(v_{oy}/g)$

For projectiles traveling over level ground, the *fall* time is the same as the rise time, as the proof below will demonstrate.

Rise Time Same as Fall Time

The rise time is the time it takes the ball's vertical velocity to be reduced to zero

$v_y = v_{oy} - gt$
$0 = v_{oy} - gt$
$t = v_{oy}/g$ (This is the rise time.)

Fall Time = Total Time - Rise Time
$= 2 (v_{oy}/g) – v_{oy}/g$
$= v_{oy}/g$
= Rise Time

Thus, the fall time is the same as the rise time.

The short list of equations below are the main ones needed to solve problems involving projectiles.

$v_x = v_{ox}$
$x = v_{ox}\, t$
$y = v_{oy}\, t - ½\, gt^2$
$v_y = v_{oy} – gt$
Rise Time = v_{oy}/g
Flight Time = $2\,(v_{oy}/g)$

Example A:

A struck golf ball leaves the club face with a vertical velocity of v_{oy} = 49 m/s, and travels upward, then downward, while moving horizontally the entire time.

The ball's initial horizontal velocity was v_{ox} = 12 m/s.

What maximum height is reached by the golf ball?

Answer:

At the instant the ball has reached maximum height, its vertical velocity v_y is zero:

$v_y^2 = v_{oy}^2 - 2gy$
$0^2 = 49^2 - 2\,(9.8)\,y$
$y = 122.50$ m

Example C:

Solve the problem in Example A by a different method.

$v_y = v_{oy} - g\,t$
$0 = 49 - 9.8\,t$
$t = 5.0$ s (this is the rise time)
$y = ½\,(v_{oy} + v_y)\,t$
$= ½\,(49 + 0)\,5.0$
$= 122.50$ m

Example B:

A stone is thrown with unknown initial vertical velocity and initial horizontal velocity v_{ox}= 200 m/s. The bullet strikes the ground 1.14 seconds later.

(a) How far horizontally did the bullet travel?

$x = v_{ox}\,t$
$= (200 \text{ m/s})(1.14 \text{ s})$
$= 228$ m

(b) What was the bullet's initial vertical velocity?

Total Flight Time = 1.14 s
Rise Time = ½ (Flight Time)
= ½ (1.14 s)
= 0.57 s
$v_y = v_{oy} - gt$
$0 = v_{oy} - (9.8 \text{ m/s}^2)\,(0.57 \text{ s})$
$v_{oy} = 5.59$ m/s

Example D:

The horizontal velocity of a projectile traveling over level ground is 12 m/s. Its rise time is 4.0 seconds.

How far horizontally will the projectile travel in 5.0 seconds?

Total time in the air:
t = 2 (Rise time)
= 2 (4.0 s)
= 8.0 s
$x = v_{ox}\,t$
= 12 m/s (8.0 s)
= 96 m

Example A:	Solution:
A bullet is fired from a pistol at a certain elevation above the ground, leaving the pistol with an initial vertical velocity of 98 m/s, and an initial horizontal velocity of 120 m/s. How far horizontally (x = ?) does the bullet travel before returning to the same elevation from which it was fired?	$t = 2\ (v_{oy}/g)$ $= 2\ (98/9.8)$ $= 20$ s (Total time) $x = (120\ m/s)\ (20\ s)$ $= 2400$ m

Example B:

A football kicked from the ground with an initial horizontal speed $v_{ox} = 25$ m/s lands on the grass 75 m away.

What was the ball's initial speed?

Horizontal Motion:

$x = v_{ox}\ t$

$75 = 25\ t$

$t = 3.0$ s (this is the total time)

Rise Time = ½ (3.0 s)

= 1.5 s

Vertical Motion:

$v_y = v_{oy} - g\ t$

$0 = v_{oy} - 9.8\ (1.5)$

$v_{oy} = 14.7$ m/s

Example:

A golf ball struck by a golf club leaves the ground with a vertical velocity $v_{oy} = 14$ m/s and a horizontal velocity $v_{ox} = 10$ m/s. How far horizontally (x =?) does the ball travel before striking the ground?

$t = 2\, v_{oy}/g$
$= 2\,(14)/9.8$
$= 2.86$ s

$x = v_{ox}\, t$
$= (10 \text{ m/s})\,(2.86 \text{ s})$
$= 28.6$ m

The Sum of 90^o Rule

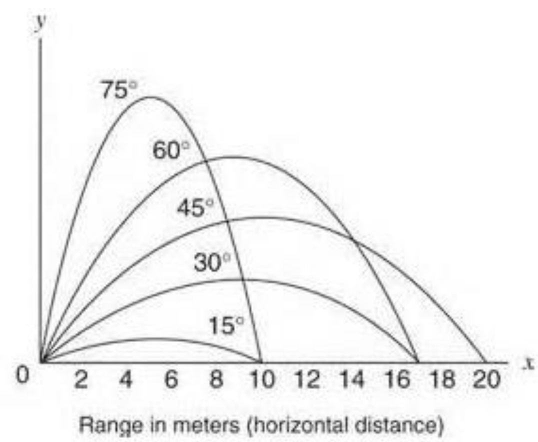
The distance a projectile travels horizontally over level ground is called its "range."

The "sum of 90^o" rule: The range of a projectile fired at an angle θ is the same as its range when fired at an angle $90 - \theta$. Examples are below

$75^o + 15^o = 90^o$
$60^o + 30^o = 90^o$
$15^o + 75^o = 90^o$

The diagram shows that a ball thrown at an angle of 15^o lands 10 meters away. If it's thrown at the same speed at an angle of 75^o, it lands the same distance away: 10 meters. Other pairs of firing angles give similar results. The 30^o - 60^o pair have a range of about 17 meters.

The greatest range is achieved when the ball is launched at an angle of 45^o with respect to the ground. In the figure, the maximum range is 20 m.



Two-Dimensional Displacements

"Headings" are directions given in degrees relative to north. For example, a heading of 90° is the same as "eastward," while a heading of 270° is "westward."

<u>Examples</u>:

20° south of east: heading of 110°
20° north of east: heading of 70°
30° west of north: heading of 330°

<u>Example</u>:

A hiker completes the displacements listed below:

A: 200 m, 270° heading ("due west")
B: 200 m, 360° heading ("due north")
C: 150 m, 90° heading ("due east")
D: 100 m, 180° heading ("due south")

How far is the hiker from her starting point?

Using the Pythagorean Theorem:

$S = (50^2 + 100^2)^{1/2}$

$= 111.80$ m

Note: The hiker could have accomplished the same thing by traveling only 111.80 m, instead of 650 m, on a heading we estimate (guess) to be about 30° "west of north."