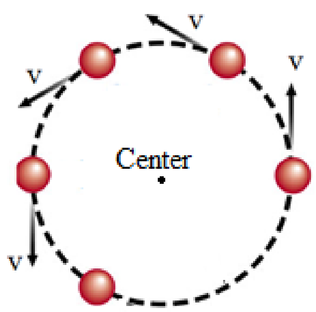
Physics 17 Part D Joseph F. Alward, PhD

Circular Motion

There are two kinds of acceleration. The kind we discussed in previous lectures was the kind that changes the object's speed.

The other kind, not previously discussed, is the one that causes a change in the object's *direction* of motion.

The figure below shows an object moving in a circular path at constant speed, v. Its continually changing direction of motion is always tangent to the circle.

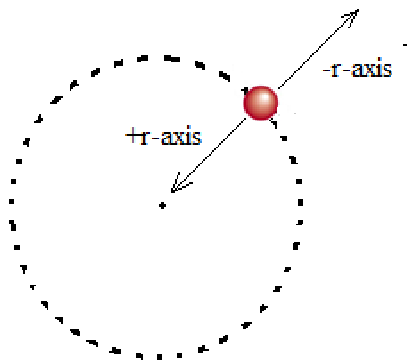


Centripetal Acceleration

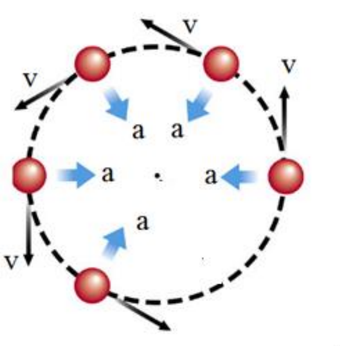
In studies of circular motion the relevant axes along which forces point are the so-called "radial" axes.

The positive radial axis (+r-axis) is inwardly-directed, pointing from the object toward the circle's center.

The negative radial axis (-r-axis) is outwardly-directed, pointing from the object, away from the center. The two radial axis are illustrated below:



Objects moving in a circular path at constant speed have a type of acceleration that doesn't change the speed, but *does* change the *direction* of motion, which is continually changing, always pointing from wherever the object is at any moment, toward the center of the circle. A few of these acceleration directions are illustrated below.



The acceleration of the object, **a**, is always directed inward, in the +r direction, and is called "centripetal acceleration." Centripetal accelerations are calculated using the equation below:

$$\mathbf{a = v^2/r}$$

Unlike accelerations in earlier parts of the course in which accelerations were sometimes positive, and other times negative, *centripetal accelerations are always positive.*

Example:

An object is swinging at constant speed in a circular path at the end of a string. The centripetal acceleration of the object is 24 m/s^2.

$a = v^2/r$
$= 24\ m/s^2$

If the speed is halved and string length tripled, what would be the new acceleration?

Halving the speed makes numerator $(1/2)^2 = 1/4^{th}$ of its previous value:

$24/4 = 6\ m/s^2$

Next, tripling the string length (tripling the denominator) makes the acceleration one-third of what it once was:

$(1/3)\ (6\ ) = 2\ m/s^2$

Newton's Second Law and Circular Motion

Inward and Outward Radial Forces

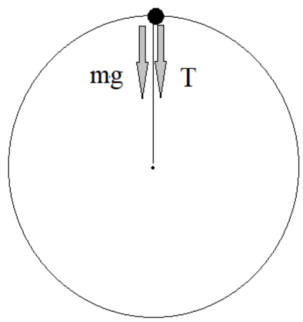
Positive radial forces are directed inward (toward the center) along the positive radial axis (+r-axis), while negative radial forces are directed outward, away from the center, along the negative radial axis (-r-axis).

In applying Newton's Second Law to circular motion, the net radial force is the sum of the inward and outward radial forces acting on the object.

m = Mass of the object
F = Sum of the radial forces
$F = m\ (v^2/r)$

Example :

A ball of mass m = 1.20 kg is moving at 4.0 m/s in a *vertical* circular path at the end of a 1.40-m string. What is the tension in the string when the ball is at the top of the swing?



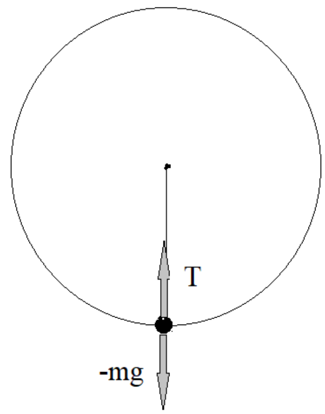
In this problem there are two forces acting on the object. Those forces are shown above: the weight force mg, and the pull by the string, T.

Both of the forces are inwardly-directed (positive) forces, pointing toward the center of the circular path. The net radial force is the sum of the two forces:

$$F = ma$$
$$T + mg = mv^2/r$$
$$T = mv^2/r - mg$$
$$= 1.20\,(4.0)^2/1.40 - (1.20)(9.8)$$
$$= 1.95\text{ N}$$

Example:

This example problem below is identical to the one above, except the ball is at the *bottom* of the circular path, not the top.



A 1.20-kg ball is moving in a vertical circular path. The ball is moving at 4.0 m/s at the bottom of a 1.40-m string. What is the tension in the string?

Solution:

The tension force points along the +r direction, and is therefore is therefore positive. The weight force is an outwardly-directed radial force, pointing in the -r direction and therefore is negative.

$$F = ma$$
$$T - mg = mv^2/r$$
$$T = mv^2/r + mg$$
$$= 1.20\,(4.0)^2/1.40 + (1.20)(9.8)$$
$$= 25.47\text{ N}$$

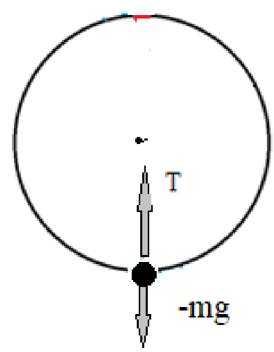
Notice how much greater is the string tension at the bottom compared to at the top.

Example:

A certain string will break when its tension exceeds 500 N. At the end of a 1.50-m length of this string an object of mass m = 2.0 kg is swinging in a vertical circular path.

The string is most vulnerable to breaking at the bottom of the swing. What is the greatest speed the object could have without the string breaking?

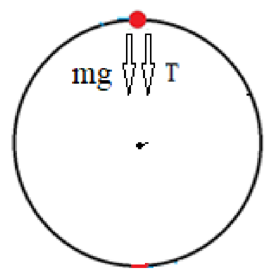
The figure below shows an object moving in a circular path at the bottom of the swing.



$$F = ma$$
$$T - mg = mv^2/r$$
$$T = mv^2/r + mg$$
$$500 = 2.0\ v^2/1.2 + 2.0\ (9.8)$$
$$v = 16.98\ \text{m/s}$$

Example:

A ball of mass m = 1.80 kg is moving at the top of a vertical circular path at the end of a 2.0-m string. At what speed would the tension be zero?



The two forces shown above are both inwardly-directed radial forces, pointing along the + r direction, so each one is positive. The total radial force is the sum of the two forces.

$F = ma$

$T + mg = mv^2/r$

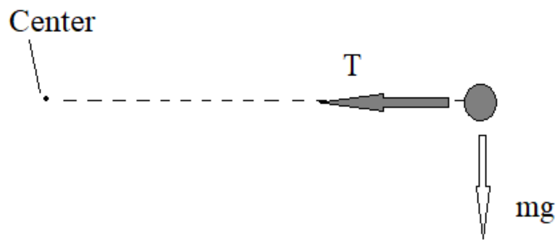
$T = mv^2/r - mg$

$0 = 2.0\ v^2/1.50 - 9.8$

$v = 3.83\ m/s$

Example:

An object of mass m = 2.0 kg is moving at the end of a string in a *horizontal* circular path of radius 0.60 m. Its speed is v = 4.0 m/s. What is the string's tension?



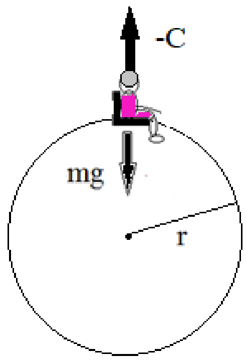
The figure above is an edge view of the plane in which the motion is taking place. It shows two forces acting on the object, tension T lying in the plane of the motion, and weight mg, perpendicular to the plane.

The tension force is an inwardly-directed radial force, and therefore is positive. The weight force is not a radial force, so the total radial force is just the tension force, T.

F = ma
$T = (2.0)\ (4.0^2)/0.60$
$= 53.33$ N

The Ferris Wheel

Example:



The radius of a Ferris wheel is r = 5.0 m. At what speed will a rider lose contact with her seat, i.e., at what speed will the contact force the seat exerts upward on her be zero?

Solution:

The weight force mg is an inwardly-directed force, pointing along the + r direction, toward the center, so it's positive. However, the contact force C is outwardly-directed, pointing along the -r direction, so it's negative.

$$F = ma$$
$$mg - C = mv^2/r$$
$$mg - 0 = mv^2/r$$

$$g = v^2/r$$
$$9.8 = v^2/5.0$$
$$v = 7.0 \text{ m/s}$$

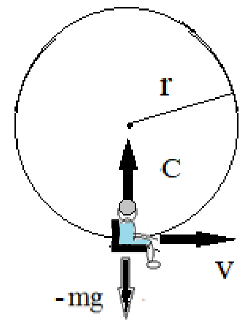
Example:

The radius of a Ferris wheel is 4.3 m. A passenger whose mass is 72 kg is moving at a speed v = 1.3 m/s. What is the passenger's apparent weight?

Solution:

The apparent weight of an object is whatever is the contact force C exerted upward on it.

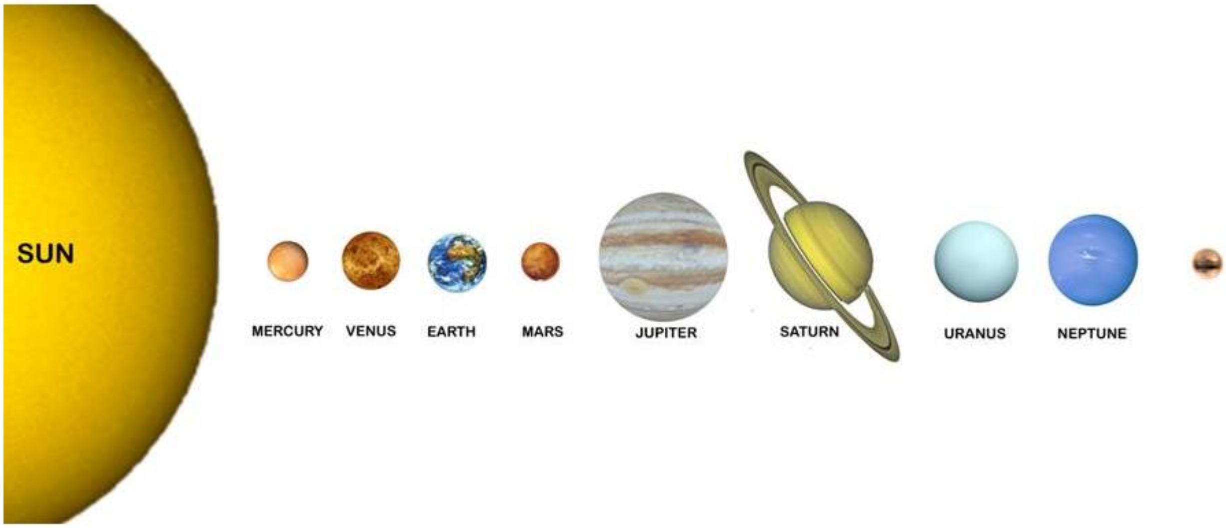
The contact force C is inwardly-directed, along the +r direction, so it's positive. The weight force mg is outwardly-directed, acting along the -r direction, so it's negative.



$$F = ma$$
$$C - mg = mv^2/r$$
$$C = mg + mv^2/r$$
$$= 72\,(9.8) + 72\,(1.3)^2/4.3$$
$$= 705.60\text{ N} + 28.30\text{ N}$$
$$= 733.90\text{ N}$$

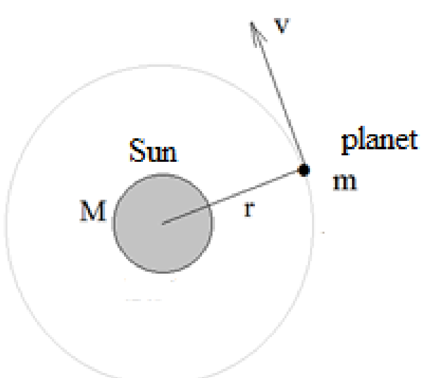
Apparent weight = 733.90 N
Actual weight = mg
= 705.60 N
The passenger's apparent weight 28.30 N more than her actual weight. She feels heavier.

Planetary Motion



Distances illustrated above are not to scale.

The planets in our solar system move in roughly circular orbits. We derive below the relationship between a planet's orbital radius **r** and its speed **v**.



Mass of Sun:
$M = 2.0 \times 10^{30}$ kg
m = Mass of Planet
$G = 6.67 \times 10^{-11}$ N-m^2/kg^2
$GM = (6.67 \times 10^{-11})\ (2.0 \times 10^{30})$
$= 1.33 \times 10^{20}\ m^3/s^2$

$$F = ma$$
$$GMm/r^2 = mv^2/r$$

$$rv^2 = GM$$
$$= 1.33 \times 10^{20}\ m^3/s^2$$

This relationship shows that the greater the distance of a planet from the sun, the smaller is its speed; when one is large, the other is small. Though the radius and speed vary from one planet to the next, the product of the two terms r and v^2 remain the same for all planets, irrespective of mass m.

Example A:

The orbital radius of Mercury is
$r = 5.8 \times 10^{10}$ m.

What is Mercury's orbital speed?

$rv^2 = GM$

$v = (GM/r)^{1/2}$
$= (1.33 \times 10^{20}/5.8 \times 10^{10})^{1/2}$
$= 47{,}886$ m/s

Example B:

How long (in days) does it take Mercury to complete one orbit around the Sun?

Mercury's orbital radius is 5.8×10^{10} m, and its orbital speed is 47,886 m/s.

Distance = Circumference
$= 2\pi\ (5.8 \times 10^{10}\ \text{m})$
$= 3.64 \times 10^{11}$ m

Time = Distance/Speed
$= (3.64 \times 10^{11}\ \text{m}) / 48{,}886$ m/s
$= 7.45 \times 10^{6}$ s

Number of seconds in a day:
(24 hrs/day)(3600 s/hr)) = 86,400 s/day

7.45×10^{6} s / 86,400 s/day = 86 days

The "orbital period" of a planet is the time it takes to complete one orbit. Therefore, the orbital "period" of Mercury is 86 days. Compare that to Earth's 365-day period.