Part D Problems

1. An object of mass m = 3.0 kg is traveling in a circular path of radius r = 1.6 m with speed v = 5.0 m/s. What is the object's centripetal acceleration?

2. An object's centripetal acceleration is currently 6.0 m/s^2. What will it be if its speed is tripled?

3. A 0.50-kg ball is swinging at the end of a 2.0-m string in a circular path in a horizontal plane. The ball makes five revolutions per second. What is the tension in the string?

4. A 0.70 kg ball is placed at the end of a 0.80-m string whose breaking strength is 900 N, and swung in a horizontal circular path. What is the least time (in seconds) the ball could take to complete a revolution without breaking the string?

5. A ball of mass m = 1.2 kg is swinging in a vertical plane in a circular path at the end of a string of length 1.6 m. What speed at the bottom will cause the tension in the string to be 80 N?

6. An object of mass m = 2.0 kg is being swung in a vertical circular path at the end of a string of length 2.3 m. When the object is at the bottom of its swing, the tension in the string is 110 N. What is the speed of the object?

7. Chairs on a Ferris wheel travel in a circular path of radius 7.0 meters. What speed will cause a passenger's "apparent weight" (the contact force, C) at the bottom to be four times her actual weight?

8. Imagine an "alternate" universe in which the gravitational force law is $F = GMm/r^3$. In that universe, on what power of the orbital radius does the speed depend, i.e., what is the exponent of the radius r in the velocity and radius equation?

9. Suppose a planet orbiting a certain star has an orbital speed of 10,000 m/s, and an orbital radius of 2×10^{11} m. A second planet orbiting the star has an orbital radius of 8×10^{11} m. What is that planet's orbital speed?

10. An object is swinging in a circular path in a horizontal plane at the end of a string, whose tension is 500 N. (a) What would be the new tension if the object's speed were doubled? (b) What if the speed was doubled and the length halved?

Solutions

1. $a = 5.0^2/1.6$
$= 15.63\ m/s^2$

3. v = distance/time
$= 5\ (2\pi r)/t$
$=5\ [(2\pi)2.0]/1.0$
$= 62.83\ m/s$
$T = mv^2/r$
$= 0.5(62.83)^2/2.0$
$= 986.9\ N$

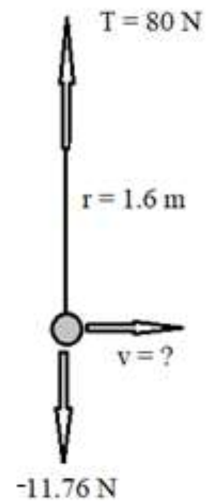
2. $a_1 = v_1^2/r$
$= 6\ m/s^2$
$v_2 = 3v_1$
$a_2 = v_2^2/r$
$= (3v_1)^2/r$
$= 9\ (v_1^2/r)$
$= 9\ (a_1)$
$= 9\ (6)$
$= 54\ m/s^2$

4. $ma = F$
$0.70\ v^2/0.80 = 900$
$v = 32.07\ m/s$

Circumference $= 2\pi(0.80)$
$= 5.03\ m$

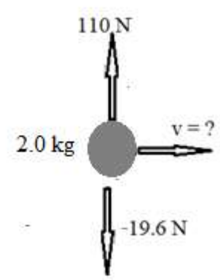
t = distance / speed
$= 5.03/32.07$
$= 0.16\ s$

5.



$80 - 11.76 = 1.2\ v^2/1.6$
$v = 9.54\ m/s$

6.



$110 - 19.6 = 2.0\ (v^2/2.3)$
$v = 10.20\ m/s$

Note: in F = ma problems, the mv^2/r term will always be positive in circular motion problems because centripetal acceleration always points in the *positive* radial direction.

7. $F = ma$
$C - mg = mv^2/r$
$4mg - mg = mv^2/r$
$4g - g = v^2/r$
$v = (3gr)^{1/2}$
$= 14.35\ m/s$

8. $GMm/r^3 = mv^2/r$
$GM = r^2v^2$
$r^2v^2 = GM$
$rv = (GM)^{1/2}$
$v = (GM)^{1/2}/r$
$= (GM)^{1/2}\ (1/r)$
$= (GM)^{1/2}\ \ r^{-1}$

The speed v depends on r to the negative one power.

Note: the “reciprocal” of x is 1/x, which is the same as x^{-1}. For example, $1/10 = 10^{-1}$.

9. $r_2 v_2^2 = r_1 v_1^2$, but $r_2 = 4 r_1$ $(4r_1)v_2^2 = r_1 v_1^2$ $v_2 = (1/4)^{1/2} v_1$ $= ½ (10,000)$ $= 5,000$ m/s
10. (a) $T_1 = mv_1^2/r$ $= 500$ N $v_2 = 2v_1$ $T_2 = m(2v_1)^2/r$ $= 4 (mv_1^2/r)$ $= 4 (500)$ $= 2000$ N Simpler: a doubling of v causes a doubling-squared (a quadrupling) of the tension. (b) $T_1 = mv_1^2/r_1$ $= 500$ N $v_2 = 2v_1$ $r_2 = ½ r_1$ $T_2 = m(2v_1)^2/(½ r_1)$ $= 8 (mv_1^2/r)$ $= 8 T_1$ $= 8 (500)$ $= 4,000$ N Simpler: a doubling of v causes a quadrupling of the tension to 2000 N, then a halving of r causes a doubling of 2000 N to 4000 N.