Physics 17 Part E

Work Power Kinetic Energy

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Let F be one of the forces acting on an object; F is negative if it points to the left or downward, and positive otherwise.

Let x be the object's displacement; x is negative if the movement is to the left or downward

The "work" done by the force F is symbolized as W, and is calculated as indicated below:

$$W = Fx$$

If the signs of the two terms, F, and x, are the same, the work done is positive; otherwise, the work done is negative.

Units: newton-meter (N-m)

1 "joule" (J) = 1 N-m

Vertical Motion

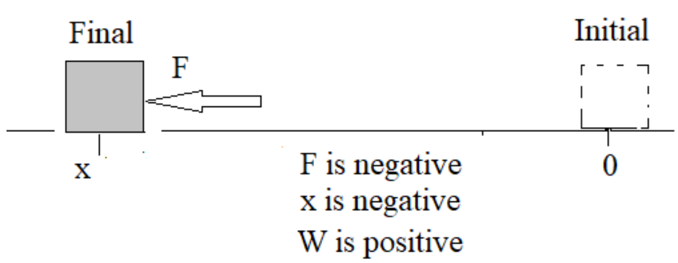
$$W = Fy$$

If the force points upward, F is positive; if the force points downward, F is negative. If the object moves upward a distance y, then y is positive; it moves downward, y is negative.

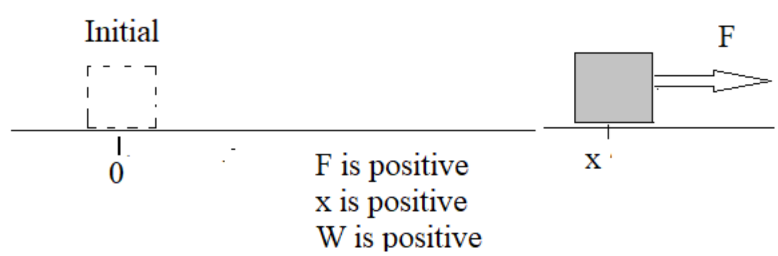
Examples of works done during vertical motion are not shown.

Three examples of work appear below The force F indicated in these diagrams may be just one of perhaps several forces (not shown) acting on the object, and the work discussed is just the work done by that single force. Other works that may have been done by other forces is not discussed.

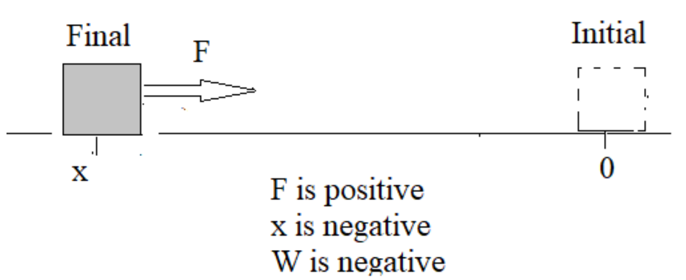
1. Object was moving to the left. One of the forces F acting on the object was pointing to the left. The work done by that force was positive.



2. Object was moving to the right. One of the forces F acting on the object was pointing to the right. The work done by that force was positive.

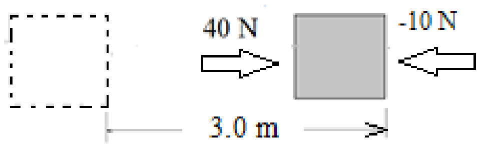


3. Object was moving to the right. One of the forces F acting on the object was pointing to the right. The work done by that force was negative.



Example A: A constant force F = 20 N is directed to the right on an object that is moving to the right. How much work does the force do when the object moves 5 m? $F = 20$ N $x = 5$ m $W = F x$ $= (20 \text{ N}) (5 \text{ m})$ $= 100$ N-m $= 100$ J	Example B: A force F = 60 N acts to the right on an object as it moves 12 meters to the left. $F = 60$ N $x = -12$ m $W = Fx$ $= (60 \text{ N}) (-12 \text{ m})$ $= -720$ N-m $= -720$ J
Example C: A 10-kg object is falling. How much work is done on the object as it falls through 8 meters? $F = -mg$ $= -10(9.8)$ $= -98$ N $x = -8$ m $W = F x$ $= (-98) (-8)$ $= 784$ J	Example D: An object weighing 100 N is thrown upward. How much work does gravity do as the object moves upward 5 m ? $F = -100$ N $x = +5$ m $W = Fx$ $= -100 (5)$ $= - 500$ J

Example:



Two forces act on the object as it moves 3.0 m to the right. What is the total work done on the object?

There are two ways to find the total work.

Add the two separate works:

$$W = 40(3.0) + (-10)(3.0)$$
$$= 120 - 30$$
$$= 90 \text{ J}$$

Or, first find the total force:

$$F = 40 - 10$$
$$= 30 \text{ N}$$

$$W = 30\,(3.0)$$
$$= 90 \text{ J}$$

Power

Machines, persons, animals, gravity, and other "agents," do work, sometimes slowly, sometimes rapidly. The greater the amount of work done, and in the shorter time, the greater is the "output power" of the agent.

The power output is the work done per second:

$$P = W/t$$

Units: J/s
Let 1 "watt" = 1 J/s

Example A:

Over a 10-second period a machine does 500 J of work. What was the machine's power output?

$P = W/t$
$= 500\ J/10\ s$
$= 50\ J/s$
$= 50$ watts

Horsepower

A non-standard unit of power commonly used is the "horsepower."

1 horsepower (hp) = 746 watts

Example:

An elevator cab has a mass of 600 kg, and contains a 500-kg cargo (six persons), for a total of 1100 kg.

What must be the power (in horsepower, hp) of the motor which would lift the cab and its contents 100 meters each minute (60 s)?

Solution:

The cable attached to the cab must exert a force F upward at least equal to the weight mg of the 1100 kg mass, in order to cause it to move upward:

$F = mg$
$= 1100\ (9.8)$
$= 10{,}780$ N
$x = 100$ m

$W = Fx$
$= (10{,}780\ \text{N})\ (100\ \text{m})$
$= 1.078 \times 10^6$ J

$P = 1.078 \times 10^6$ J/ 60 s
$= 1.80 \times 10^4$ J/s
$= 1.80 \times 10^4$ watts

What is this power in *horsepower* units?

$(1.80 \times 10^4$ watts) / (746 watts/hp) = 24.1 hp

Another Way to Calculate Power

$P = W/t$

Substitute Fx for W:

$P = (Fx)/t$
$\quad = F\,(x/t)$

Recall: $\bar{v} = x/t$

$$P = F\bar{v}$$

Example :

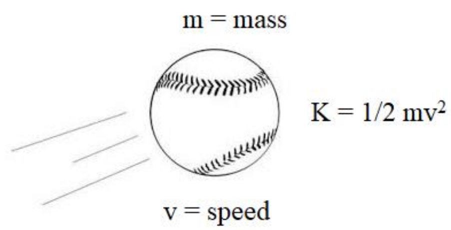
A crane's output power is 10 hp as it lifts a 10,000 N object. At what average speed $\bar{v}$ will the object move upward?

$P = 10$ hp
$\quad = 10\ (746)$
$\quad = 7460$ watts

$F = 10{,}000$ N

$P = F\bar{v}$
$7460 = 10{,}000\ \bar{v}$
$\bar{v} = 0.746$ m/s

Kinetic Energy



All moving objects have a type of "energy" called "kinetic," from the Greek word for motion, "kinesis."

Unlike displacements and velocity, kinetic energy is not a directional quantity. The kinetic energy of an object depends only on the speed (absolute value of velocity) of an object, and its mass, neither of which are directional quantities.

The symbol for kinetic energy is K:

$$K = \frac{1}{2} mv^2$$

<u>Example</u>:

What is the kinetic energy of a 4 kg object moving at 5 m/s?

$K = \frac{1}{2}(4\text{ kg})(5\text{ m/s})^2$
$= 50\text{ kg-m}^2/\text{s}^2$
$= 50\text{ (kg-m/s}^2\text{) m}$
$= 50\text{ N-m}$ Recall: 1 joule (J) = 1 N-m
$= 50\text{ J}$

The units of kinetic energy are the same as the units of work: joules (J)

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Example A:	Example B:
$K = \frac{1}{2} mv^2$ The kinetic energy of an object is 400 J. What will be the change in the object's kinetic energy if its speed is halved? If v is halved, v^2 is quartered, which means that K will also be quartered, down to 100 J. $\Delta K = K - K_o$ $= 100 - 400$ $= -300$ J	The kinetic energy of an object is 300 J. What will be the change in its kinetic energy if its speed is tripled? If v is tripled, then v^2 increases to nine times its previous value: $K = 9\,(300)$ $= 2700$ J $\Delta K = 2700 - 300$ $= 2400$ J

The Work-Kinetic Energy Theorem

Recall $v^2 = v_o^2 + 2ax$

Multiply by ½ m:

$\frac{1}{2} mv^2 = \frac{1}{2} mv_o^2 + (ma)\,x$

Recall F = ma, where F is the total force.

Replace ma in Equation 1 with F:

$$\frac{1}{2} mv^2 = \frac{1}{2} mv_o^2 + Fx$$
$$= \frac{1}{2} mv_o^2 + W$$

Note: F is the total force, so W is the total work.

$$\frac{1}{2} mv^2 - \frac{1}{2} mv_o^2 = W$$
$$K - K_o = W$$
$$\Delta K = W$$

or

$$W = \Delta K$$

The Work-Kinetic Energy Theorem is stated below:

"The total work done on an object equals the change in its kinetic energy."

Example:

How much work would have to be done to increase a 6-kg object's speed from 3 m/s to 8 m/s?

$W = K - K_o$
$W = ½ (6)8^2 - ½ (6)3^2$
$= 165$ J