Physics 17 Part F

Gravitational Potential Energy

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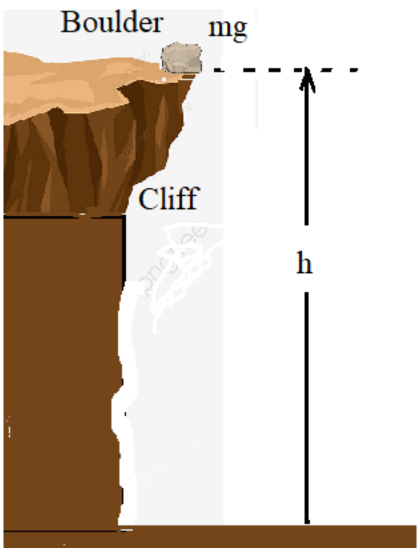
A boulder resting precariously at the edge of a cliff has the future capability (the "potential") for doing work on the ground below after it tips over the edge, accelerates due to gravity, and strikes the valley ground below, doing work on it by creating a crater. The landing point need not necessarily be the valley ground floor; perhaps the boulder lands on a ledge half-way up the cliff. In such as case, the boulder's speed will be less, so it will do less work than it would if it falls a greater distance. In the ledge scenario, the ledge will serve as the "ground" for the purpose of measuring heights of



The more massive the boulder, and the higher up it is above the reference ground, the more work it can do when it lands on the ground. The work Earth does on the falling boulder equals Earth's pull mg on it, times the distance x of fall, where x is the boulder's initial height h above the reference ground:

$$W = mgh$$

By Newton's Third law, the work Earth does on the boulder equals the work the boulder does on Earth (the ground). We define the "gravitational potential energy" of any object to be the work it will do when it strikes the ground, and symbolize that energy as U:

$$U = mgh$$

<u>Example</u>:

A boulder whose mass is 70 kg rests atop a cliff 50 meters high. (a) What is the boulder's gravitational potential energy? (b) How much work can it do in the future?

mg = 70 (9.8)
= 786 N

(a) U = (786 N) (50 m)
= 39,300 N-m (N-m are work units, same as joules (J).
= 39,300 J

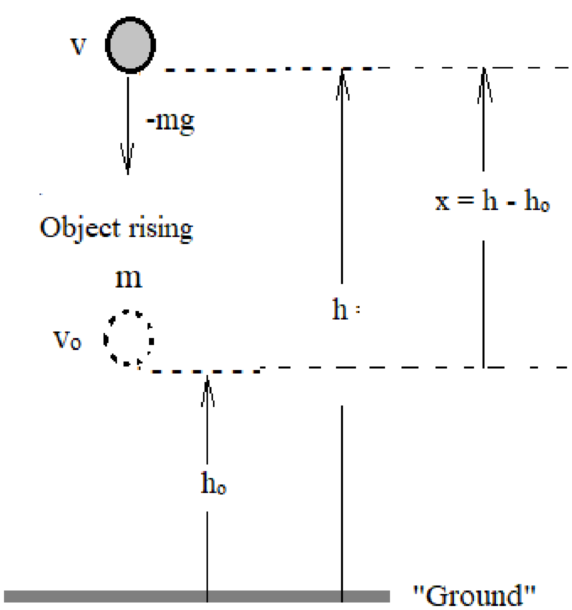
(b) Work it can do: 39,300 J

The Law of Conservation of Energy

Consider an object traveling upward. It's initial height above the reference ground (this "ground" may be the top of a builder, or the actual ground below, or any other "ground" the user may wish to specify) is h_o, and its speed at that point is v_o. Later, its height is h, and its speed at that point is v.

The displacement of the object is the distance the object travels upward from its initial height:

$$x = h - h_o$$



Relating to the figure at the left, define the quantities below:

Initial Kinetic Energy: $K_o = ½ mv_o^2$
Final Kinetic Energy: $K = ½ mv^2$
Initial Potential Energy: $U_o = mgh_o$
Later Potential Energy: $U = mgh$

Initial Total Energy: $E_o = K_o + U_o$
Final Total Energy: $E = K + U$

Apply the Work-Kinetic Energy Theorem to the rising object shown at the left:

$$\Delta K = W$$
$$= F x$$
$$½ mv^2 - ½ mv_o^2 = (-mg)(h - h_o)$$

Rearrange the equation above as shown below:

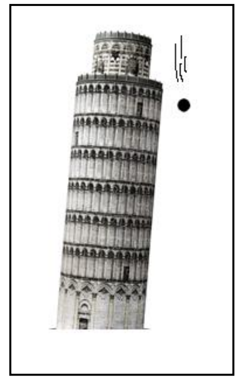
$$½ mv^2 + mgh = ½ mv_o^2 + mgh_o$$
$$K + U = K_o + U_o$$
$$\mathbf{E = E_o}$$

$E = E_o$ is the "Law of Conservation of Energy."

Total energy doesn't change over time.

Example:

A ball of mass m = 10 kg is dropped ($v_o = 0$) from a tower 20 meters tall.



(a) What is the initial total energy of the ball?

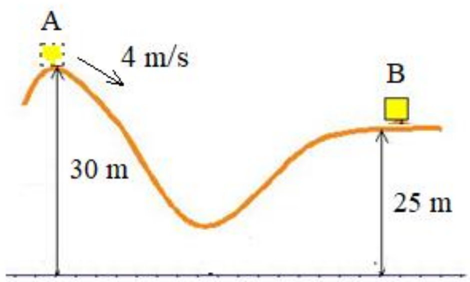
$$
\begin{aligned}
E_o &= K_o + U_o \\
&= \tfrac{1}{2} m v_o^2 + m g h_o \\
&= 0 + 10\,(9.8)20 \\
&= 1960 \text{ J}
\end{aligned}
$$

(b) What will be its speed when its potential energy is 1200 J?

$$
\begin{aligned}
E &= E_o \\
\tfrac{1}{2}(10)v^2 + 1200 &= 1960 \\
v &= 12.33 \text{ m/s}
\end{aligned}
$$

Total energy is conserved not only for objects rising and falling *straight* upward and downward, but also for objects moving horizontally while moving vertically, as is the case for projectile motion, or carts moving along a roller-coaster, or skiers moving up and down snow-covered hills.

Example A:



An object at Point A is initially moving at 4.0 m/s on a frictionless hill 30 meters high.

What will be its speed when it reaches the top of the 25-meter-tall hill at B?

$\frac{1}{2} mv^2 + mg(25) = \frac{1}{2} m(4.0)^2 + mg(30)$

Divide out the m's and solve for v:

$$v = 10.68 \text{ m/s}$$

Non-Conservative Work

Work done by air resistance or friction doesn't allow the total energy to be conserved; we speak of that work as being "non-conservative work," and symbolize that work as W_{NC}.

$E = E_o + W_{NC}$

<u>Example:</u>

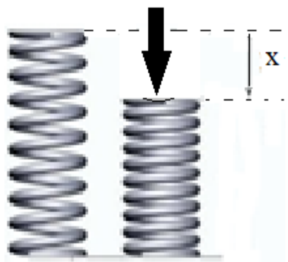
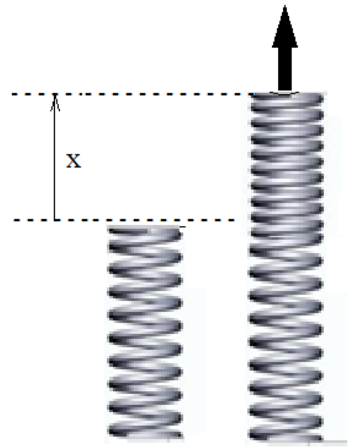
Suppose in the previous example, where the final speed was 10.68 m/s, the actual speed is 9.24 m/s, instead. What would have been the work done by air resistance? Assume the object's mass is 2.0 kg.

$E_o = ½ mv_o^2 + mgh_o$
$= ½ (2.0)(4.0)^2 + 2.0(9.8)(30)$
$= 604.00$ J

$E = ½ mv^2 + mgh$
$= ½ (2.0)(9.24)^2 + 2.0(9.8)(25)$
$= 575.38$

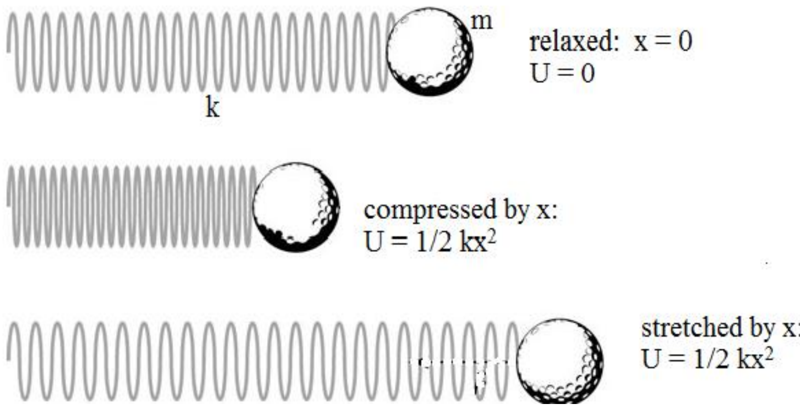
$W_{NC} = E - E_o$
$= 575.38 - 604.00$
$= -28.62$ J

Spring Potential Energy

Compressed Spring	Stretched Spring
 k = spring constant Units: N/m $U = \frac{1}{2} kx^2$ <u>Example A</u>: k = 2000 N/m (a) What is the potential energy stored in this spring when it's *compressed* by 0.40 m? $U = \frac{1}{2} (2000)(-0.40)^2 = 160$ J	 $U = \frac{1}{2} kx^2$ <u>Example B</u>: k = 2000 N/m What is the potential energy stored in this spring when it's *stretched* by 0.40 m? $U = \frac{1}{2} (2000)(-0.40)^2 = 160$ J The energy stored in a spring is the same whether it's stretched, or compressed.

Spring-Mass Systems

A "spring-mass system" is a spring with an object attached to one end of the spring.



Note that the potential energy is "symmetrical": it is the same whether the spring is stretched or compressed.

<u>Example</u>: An object of mass m = 20 kg attached to a spring lying on a frictionless tabletop was set in motion horizontally by pulling it to the right and releasing it; the object oscillates back and forth.

At a certain moment, the object's kinetic energy is 500 J, while the spring potential energy is 900 J.

(a) What is the total energy of the system?

$E = 500 + 900$

$= 1400$ J

(b) What will be the kinetic energy of the object when the spring potential energy is 300 J?

$K + U = 1400$ J

$K = 1400 \text{ J} - 300 \text{ J}$

$= 1100$ J

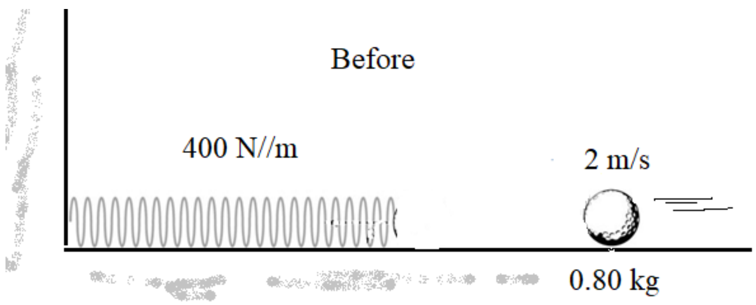
(c) What will be the speed of the object in Part(b)?

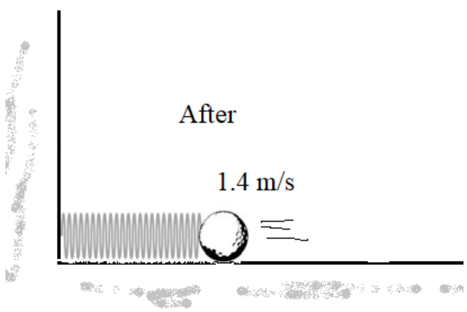
$\frac{1}{2}(20)v^2 = 1100$

$v = 10.49$ m/s

Example:

A 0.80 kg object moving at 2 m/s collides with a relaxed spring whose spring constant is 400 N/m. By how much is the spring compressed when the object's speed has been reduced to 1.4 m/s?





$$\tfrac{1}{2}(400)\,x^2 + \tfrac{1}{2}(0.80)(1.4^2) = \tfrac{1}{2}(0.80)(2^2)$$

$$x = 0.064\text{ m}$$