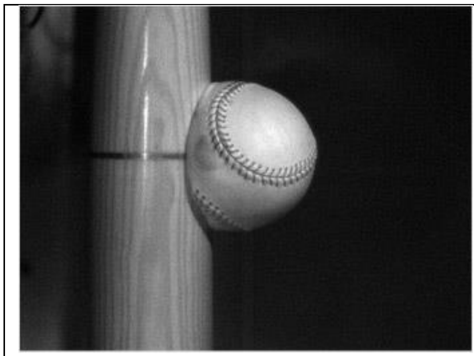
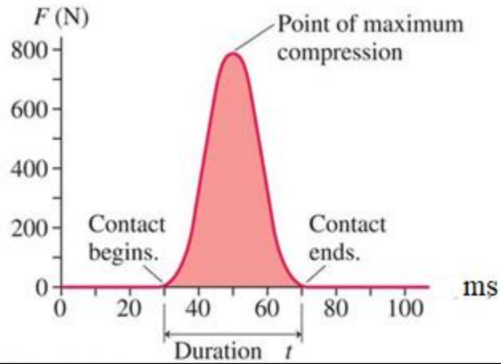
Impulse





Contact Time

The amount of time the ball is in contact with the bat in the figure above is symbolized as t, and is called "the contact time." During this time, the force the bat exerts on the ball rises from zero, reaches some maximum value, and then falls to zero when the ball breaks contact with the bat. The graph indicates a clock was started at time 0 milliseconds (ms), and that the contact began at about 30 ms, and contact ended when the clock reading was about 70 ms. The contact time period lasted about 40 ms (0.040 seconds):

$$t = 0.040 \text{ s}$$

The maximum force was about 800 N, but forces in the upper range 400-800 N existed for less time than those forces in the range 0-400 N. The average force shown in the figure above may be estimated to be about 200 – 400 N. Let's just say the average was 300 N.

The symbol for impulse is I, and is defined to be the product of the average force and the contact time:

$$I = \bar{F}\, t$$

The impulse delivered to the ball by the bat is

$$I = 300 \text{ N } (0.040 \text{ s})$$
$$= 12.0 \text{ N-s}$$

The SI units of impulse are newton-seconds (N-s).

Example A:

An impulse of 20 N-s is applied over a period of 0.04 s to an object.

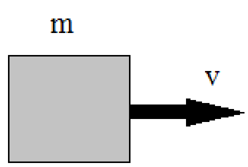
What average force was exerted on the object during this time period?

Solution:

$I = \bar{F} t$

$\bar{F} = I / t$
$= 20 \text{ N-s}/0.040 \text{ s}$
$= 500 \text{ N}$

Momentum



The momentum of an object is the product of its mass, and velocity. Momentum is often symbolized as "p":

$$p = mv$$

Most of the time we will use the product "mv" to symbolize the momentum, not "p."

Example B:

An object of mass m = 50 kg is moving with velocity v = 30 m/s.

What is its momentum?

p = mv
= 50 kg (30 m/s)
= 1500 kg-m/s

The SI units of momentum are kg-m/s.

Same as:
kg-m/s = [kg-m/s^2] s
= [N] s

Momentum units are the same as impulse units.

Impulse-Momentum Theorem

We obtain below the relationship between the total impulse delivered to an object and the change in the object's momentum.

We begin the proof with a recall of two important equations dealing with the *total* force and the resulting acceleration; these equations appeared in an earlier part of this course in our introduction to Newton's Second Law:

$$\bar{F} = m\bar{a}$$
$$\bar{a} = (v - v_o)/t$$

Insert the second expression into the first equation, and rearrange:

$$\bar{F}t = mv - mv_o$$

The equation above is the "Impulse-Momentum Theorem" which expresses the fact that the change in momentum of an object equals the total impulse delivered to it, and vice-versa.

The impulse above is the *total* impulse because the average force above is the *total* average force.

Below is stated the impulse-momentum theorem:

The total impulse an object experiences equals its change in momentum.

Example A:

A 1200-kg car moving to the right at 30 m/s crashes into a tree and comes to rest after 0.02 second. What average force did the tree exert on the car?

$\bar{F}t = mv - mv_o$

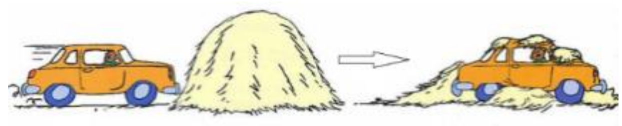
$\bar{F} = [1200\ (0) - 1200(30)]\ (\text{kg-m/s}) / 0.02\ \text{s}$

$= -1.8 \times 10^6\ \text{kg-m/s}^2$
$= -1.8 \times 10^6\ \text{N}$

Note: The negative sign indicates that the force on the car is directed to the left, as expected.

Example B:

Suppose the car in Example A crashes into a haystack instead of the tree and comes to rest eventually after 2 seconds.



What average force did the haystack exert on the car?

$\bar{F}t = mv - mv_o$
$\bar{F}(2) = 1200(0) - 1200\ (30)$
$\bar{F} = -1.8 \times 10^4\ \text{N}$

The haystack exerts an average force one-hundred times smaller than the tree's force.

Example C:

A 10-kg object moving to the right at 6.0 m/s experiences a net impulse of -14.0 N-s.

What is the object's velocity (in m/s) at the end of the impulse?

$I = mv - mv_o$
$-14.0\ \text{kg-m/s} = (10\ \text{kg})\ v - (10\ \text{kg})\ (6\ \text{m/s})$
$v = 4.6\ \text{m/s}$

Maximizing the Distance of a Throw: Follow-Through

$\bar{F}$ is the average force applied to the ball by the hand of the thrower, and **t** is the amount of time the hand is pushing on the ball. During this time, the ball's velocity changes from $v_o = 0$, to some final velocity at the end of the contact time.

$$\bar{F}t = mv - mv_o$$
$$= mv - 0$$
$$= mv$$

$$v = \bar{F}t/m$$

To maximize the distance of a throw, do two things:

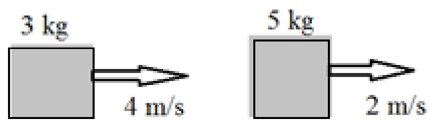
1. Throw at an angle of 45^o above the ground.
2. Throw as hard as you can, i.e., large force F.
3. Throw for a "long time," i.e., make the contact time (t) long.

Maximizing the contact time is accomplished by rotating the body counter-clockwise (for a right-hander), and leaning in the direction during the throw. This action allows the right arm and hand to "chase" the ball, i.e., delay the coming of the time at which the ball escapes from the hand and is moving on its own, thereby prolonging the hand-to-ball contact and acceleration time. Longer acceleration times give greater final speeds. This "chasing" action is called "follow-through," an effect that's desirable not only in throwing, but in other athletic endeavors as well, such as swinging tennis rackets and golf clubs, and throwing punches in a boxing ring.

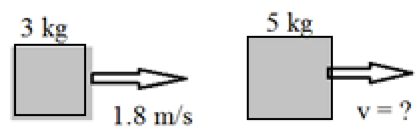
Conservation of Momentum

Without proof, we declare here that the sum of the momentums of two objects before collision equals the sum of the momentums after collision.

Example:



Before Collision

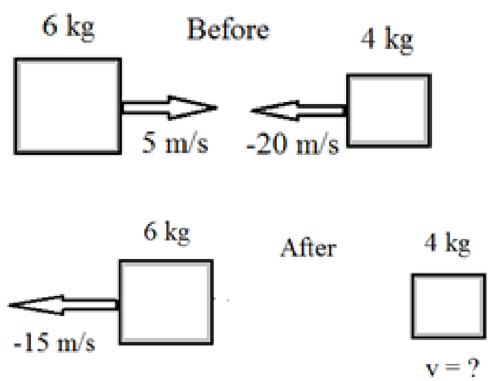


After Collision

We use upper-case "P" to represent the total momentum in a system consisting of two colliding objects:

$$P = P_o$$
$$3(1.8) + 5v = 3(4) + 5(2)$$
$$v = 3.32 \text{ m/s}$$

Example:

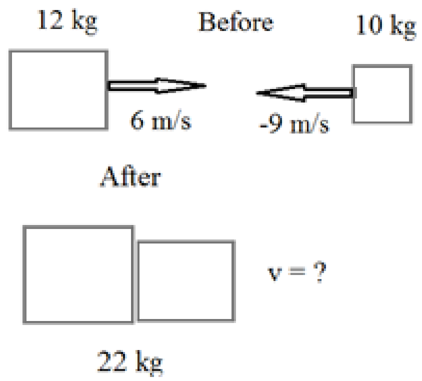


What is the velocity of the 4-kg object after collision?

$$P = P_o$$
$$6(-15) + 4\,v = 6(5) + 4(-20)$$
$$v = 10 \text{ m/s}$$

Example A:

The objects below collide and stick together.



$P = P_o$

$22 v = 12(6) + 10(-9)$

$v = -0.82$ m/s

The combined pair is moving to the left after collision, as indicated by the negative sign.

Example B:

A 2-kg wrench in the hand of a 210-kg spacewalker initially at rest is thrown to the right with velocity 5.0 m/s.

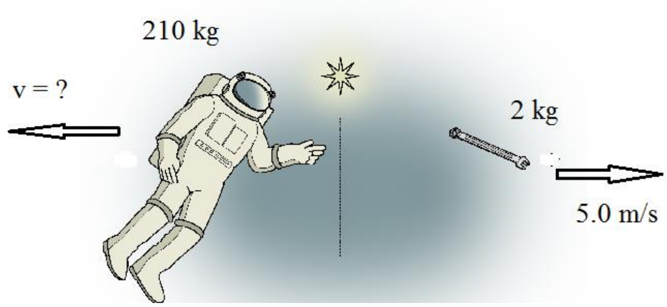
What velocity does the spacewalker acquire?

$P_o = 0$

$P = 2(5) + 210 v$

$P = P_o$

$v = -0.05$ m/s



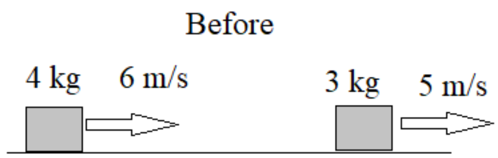
The reason the spacewalker moves to the left is because when she pushes the wrench to the right, the wrench pushes on her to the left, by Newton's Third Law.

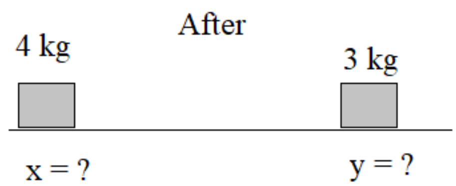
Elastic Collisions

If the total kinetic energy of colliding objects is conserved, the collision is "elastic." If the total kinetic energy is not conserved, the collision is called "inelastic."

Example:

Assume the collision below is elastic:





Find the velocities x, and y.

$$P = P_o$$
$$4x + 3y = 4(6) + 3(5)$$
$$y = (39 - 4x)/3$$

$$K = \tfrac{1}{2}(4)\,x^2 + \tfrac{1}{2}(3)\,y^2 = \tfrac{1}{2}(4)\,6^2 + \tfrac{1}{2}(3)\,5^2$$
$$4x^2 + 3y^2 = 219$$

Replace y above by the expression above:
$$4x^2 + 3\,[(39 - 4x)/3]^2 = 219$$

$x = 5.14$ m/s and $y = 6.14$ m/s
or
$x = 6.00$ m/s and $y = 5.00$ m/s

We reject the solution pair, $x = 6.00$ m/s and $y = 5.00$ m/s, as nonsense. These velocities would mean that the 4-kg object supernaturally passes through the 3-kg object without either velocity changing; this cannot happen.