Physics 17 Part N

Waves

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Video Lecture 1: String Waves Intro
Video Lecture 2: Standing String Waves
Video Lecture 3: Intro to Sound Waves
Video Lecture 4: Doppler Effect for Sound
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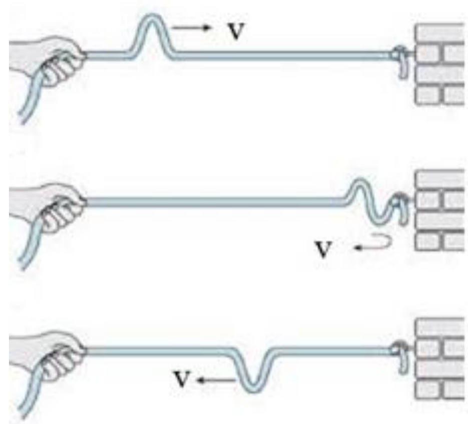
Waves on Strings



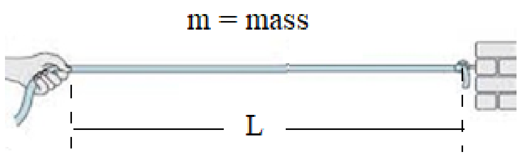
Shake once: An incident “pulse” (disturbance) is created that travels along the string; the pulse reflects back. Reflected pulses overlap (not shown) incident pulses, and if the timing is just right, the disturbances are amplified, giving rise to what is called a “resonance.”

An example of a resonance is shown below:

Explanation and examples of resonances will be provided later.

Before discussing resonances, let's look at certain string properties and behaviors that govern the creation and propagation of string waves.

Pulse Speed



L = Length of String
m = Mass
μ = Linear Mass Density
  = m/L

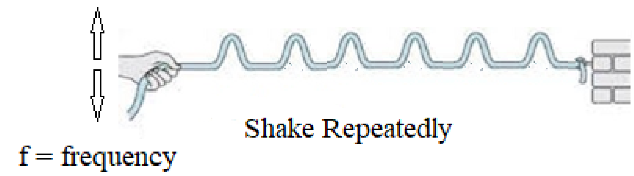
T = Tension
v = Pulse Speed
  $= \sqrt{(T/\mu)}$

<u>Example</u>:

T = 4.9 N
μ = 0.10 kg/m

$v = (4.9/0.10)^{1/2}$
  = 7.0 m/s

Wave Frequency



Repeated shakes (oscillations) of the string cause a “train” of pulses, called a “wave.”

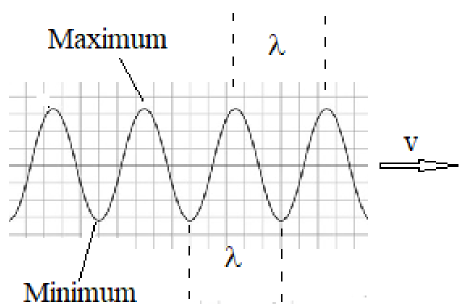
The frequency of a string wave is the number of pulses delivered to the string each second.

The units of frequency are “hertz” (Hz).

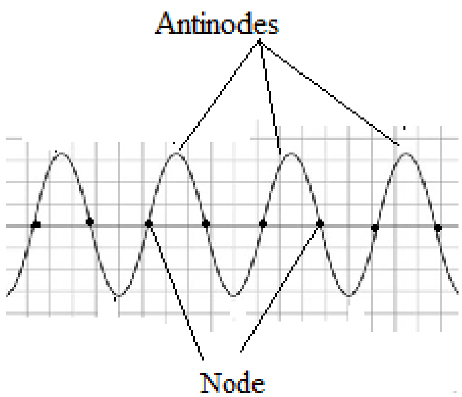
1.0 Hz = 1.0 s^{-1} (per second)

Wavelength

The wavelength of a wave is the distance between consecutive maxima, or two consecutive minima.

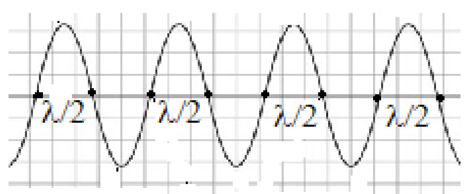


Nodes and Antinodes



Nodes are places where there is zero displacement of the string particles. A more precise name for this location is "displacement node."

"Antinodes" are places where there is maximum displacement. Antinodes occur between nodes.



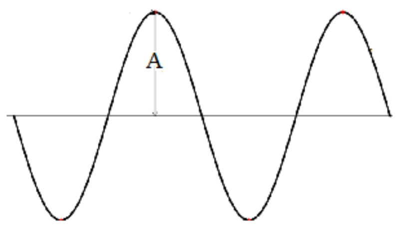
For later use, let's note here that the distance between consecutive nodes is λ/2.

At ends of strings that are tied down, there is no possibility of displacement, so those ends of the string are nodes.

Amplitude

The amplitude of a string wave is the absolute value of the string particles' maximum displacement.

The symbol for a wave's amplitude is A, a positive number.



The Wave Equation

$$\lambda f = v$$

<u>Example A</u>:	<u>Example B</u>:
The speed of waves on a string is 12.0 m/s. A t what frequency will the wavelength of a wave be 4.0 m? $f = v/\lambda$ $= (12.0\ \text{m/s})/(4.0\ \text{m})$ $= 3.0\ \text{s}^{-1}$ $= 3.0\ \text{Hz}$	$T = 100\ \text{N}$ $m = 0.4\ \text{kg}$ $L = 1.6\ \text{m}$ $f = 10\ \text{Hz}$ $\lambda = ?$ $\mu = 0.4/1.6$ $= 0.25\ \text{kg/m}$ $v = (T/\mu)^{1/2}$ $= (100/0.25)^{1/2}$ $= 20\ \text{m/s}$ $\lambda = v/f$ $= (20)/(10)$ $= 2\ \text{m}$

As we've done so many times in the past, we've omitted the units in a few of the intermediate steps of Example B, but we did so with the full confidence that as long as we use SI units everywhere, the final answer will be in SI units.

Resonances (Standing Waves)

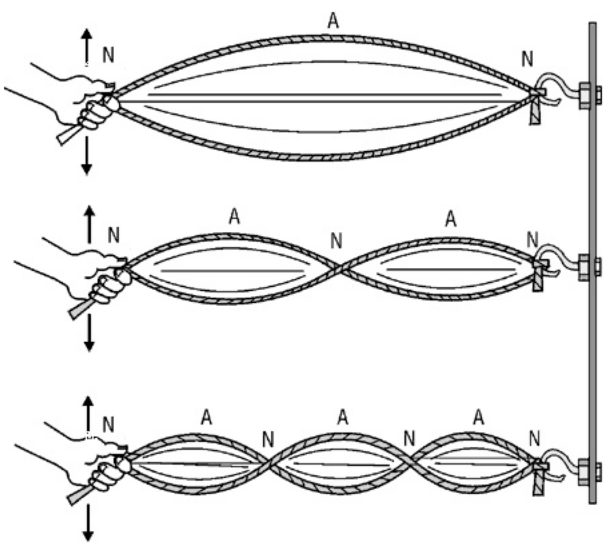
At certain frequencies a "resonance" occurs in which comparatively small amplitude vibrations* at the left end of the string below leads to comparatively large vibrations--antinodes (A)--at certain points along the string, and zero vibrations at other points--nodes (N).

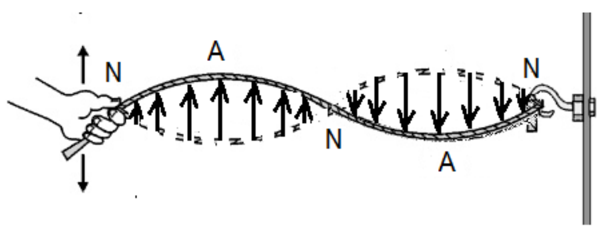
Such resonances are often called "standing waves" because certain points (nodes) on the string are not moving.

Examples of standing wave (resonances) at the left, below.

**The oscillations applied at the left end of the strings below have such small amplitude that the left ends are treated as if they were a nodes.*

The resonances below are in the shape of a series of "loops." At the ends of each loop are nodes. Recall that the distance between consecutive nodes in a wave is $\lambda/2$. Thus, each loop has a width of half of a wave-length ($\lambda/2$).





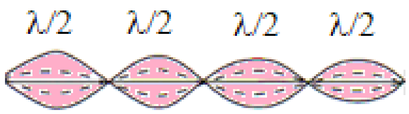
String particles in adjacent loops are oscillating "out of phase": If the string in one loop is moving upward, the string in adjacent loops is moving downward, and vice-versa.

[Click Here](#) to view video demonstration of standing waves.

Example:

The speed of pulses on a string of length 1.40 m is 2.60 m/s.

What frequency of oscillation will create a standing wave with four antinodes (four loops), as shown below?



Each of the four loops has a width of $\lambda/2$. The sum of these widths equals the length of the string*

$$4\,(\lambda/2) = 1.40$$
$$\lambda = 0.70 \text{ m}$$

$$f = v/\lambda$$
$$= 2.60/0.70$$
$$= 3.71 \text{ Hz}$$

**The "bulge" of the loops is exaggerated. If such large bulges actually occurred, the sum of the lengths of the string encompassing each loop would be significantly longer that the straight-line string length specified. For very small amplitudes, though, the difference is lengths is negligibly small, so the sum of the loop widths rules is accurate.*

Example:

Resonance on a 1.60-m string occurs with six antinodes when the string is oscillated at 6.0 Hz (6.0 s^{-1}). The tension in the string is 14.0 N. What is the linear mass density of the string?

Solution:

$6(\lambda/2) = 1.60$
$\lambda = 0.53$ m

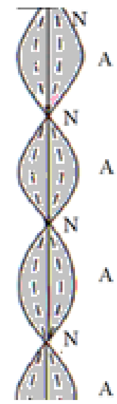
$v = \lambda f$
$= (0.53 \text{ m})(6.0 \text{ s}^{-1})$
$= 3.18$ m/s

$(T/\mu)^{1/2} = v$
$(14.0/\mu)^{1/2} = 3.18$
$\mu = 1.38$ kg/m

Resonances on Hanging Ropes

Resonances on strings can occur without both ends being tied down like the strings in our earlier examples.

For example, if the top of a rope held vertically is oscillated at small amplitude at just the right frequency, a resonance will occur.



What makes this type of resonance different from the ones studied earlier is that there are not a whole number of complete loops, but a fractional number of loops: 1.5, 2.5, 3.5 loops, and so-on.

<u>Example</u>:

A rope 0.90 meters long is hanging vertically; the bottom of the rope is free. An oscillator attached to the top of the rope is vibrating at a frequency of 4.0 Hz, which causes a resonance with four antinodes (3.5 loops).

What is the speed of waves on this rope?

Solution:

$$3.5\,(\lambda/2) = 0.90$$
$$\lambda = 0.51 \text{ m}$$
$$v = \lambda f$$
$$= 0.51\,(4.0)$$
$$= 2.1 \text{ m/s}$$

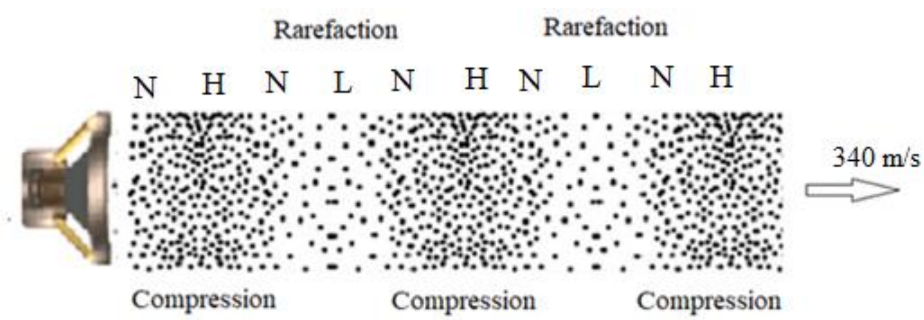
Sound Waves

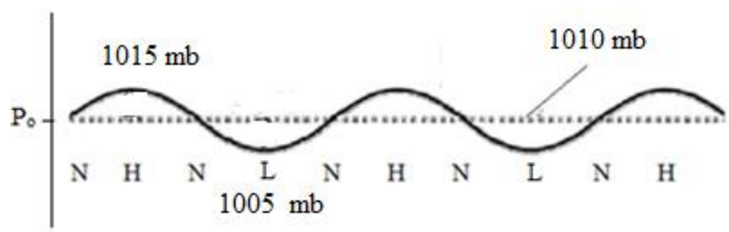
In this section, it will be worth remembering from our studies of air pressure in an earlier part of this course that atmospheric pressure at sea-level is 1010 millibar (mb).

Click the link below to gain a sense of how sound waves are created and propagate.

Video

The figures below show the "compressions" and "rarefactions" of a sound wave. The amplitude of the sound wave is 5 millibar (mb). A compression is a place where the air density and pressure is higher than normal, and a rarefaction is a place where the air pressure and density is lower.





H: High Pressure (1015 mb)
L: Low Pressure (1005 mb)
N: Normal Pressure (1010 mb)

Waves Interfering with Matter

Figure 1

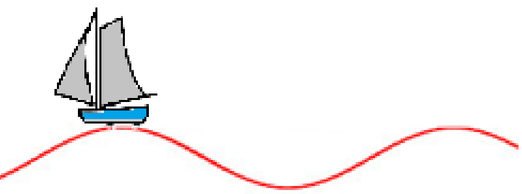
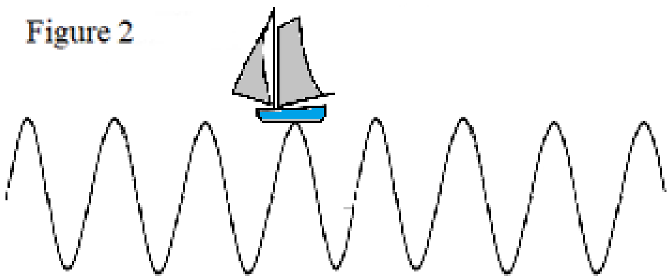


Figure 2



Waves whose wavelength are comparable to the length of objects over or through which the wave is traveling may shake the object apart. Longer wavelengths will not respond to the object's presence, nor will the object be aware of the wave see it; the wave doesn't interfere with the object, and vice-versa.

Consider the example of a ship sailing on rough versus calm seas.

In Figure 1, the wave rises and falls slowly. The distance between consecutive wave crests (the "wavelength") is many times the width of the ship: The ship is scarcely aware of the presence of the wave.

However, in Figure 2 the ship's length is comparable to wavelength. The ship abruptly rises and falls violently and is at risk of breaking apart.

The following general rule applies to objects that are candidates for disintegration by ultra-sound bombardment: the optimum wavelength of sound is one that is comparable to the diameter of the object.

Example:

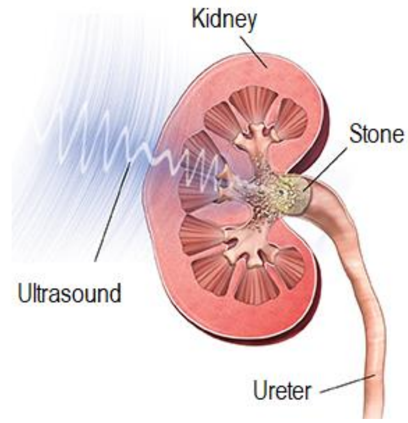
"Lithotripsy" is a medical procedure in which ultrasound is used to break up kidney stones.

What is the optimum ultrasound frequency for breaking up a kidney stone whose diameter is one centimeter (0.01 m)?

In kidney tissue, the speed of sound is about 1500 m/s.

$f = v/\lambda$
$= 1500/0.01$

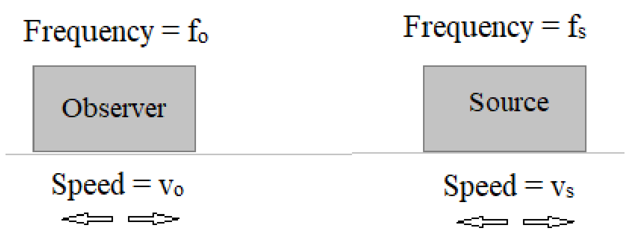
$= 150{,}000$ Hz
$= 150$ kilohertz



The Doppler Effect for Sound

Doppler effect video.

The listener hears a higher frequency as the car approaches. As it is moving away, the listener hears a lower frequency.



$$f_o = f_s \frac{(340 \pm v_o)}{(340 \pm v_s)}$$

For use below, we will refer to the numerator above as the "speed term," and the denominator as the "source term."

<u>Doppler Effect Equation Rules</u>:

<u>Approach-Larger Rule:</u> If either one is approaching the other one, choose the sign in the speed term that makes the ratio <u>larger</u>.

<u>Away-Smaller Rule</u>: On the other hand, if either one is moving <u>away</u> from the other, choose the sign in the speed term that makes the ratio <u>smaller</u>.

Example A:

Suppose an observer is at rest, and the source is moving away from the observer at 30 m/s. If the siren's frequency is 5,000 Hz, what does the observer hear?

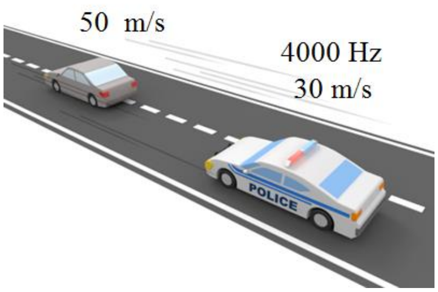
$v_o = 0$

Source is moving away, so we choose the sign in the source's speed term that will make the ratio smaller: We choose the positive sign.

$f_o = 5000\ (340 + 0) / (340 + 30)$
$= 4595$ Hz

Example B:

The police car below is chasing a speeder. What frequency does the speeder hear?



The speeder is the observer, and is moving away from the police car, so we choose the sign in the speeder's speed term that makes the ratio smaller: We *subtract* 50, not add.

The police car is moving toward the speeder, so we choose the sign in the police car's speed term that makes the ratio larger: We subtract 30.

$f_o = 4000\ (340 - 50) / (340 - 30)$
$= 3742$ Hz

Example:

A fire truck emitting 2500 Hz and traveling east at 40 m/s is racing toward an automobile traveling to the west. The automobile driver hears 3000 Hz.

What is the automobile's speed?

The observer and the source are each traveling toward each other, so we choose the sign in each speed term that makes the ratio larger.

$3000 = 2500 (340 + v_o) / (340 - 40)$

$v_o = 20$ m/s

Sound Wave Resonances

Certain sound wave frequencies resonate in tubes.

At open ends of pipes the air is free to oscillate with maximum amplitude: these are the displacement antinodes.

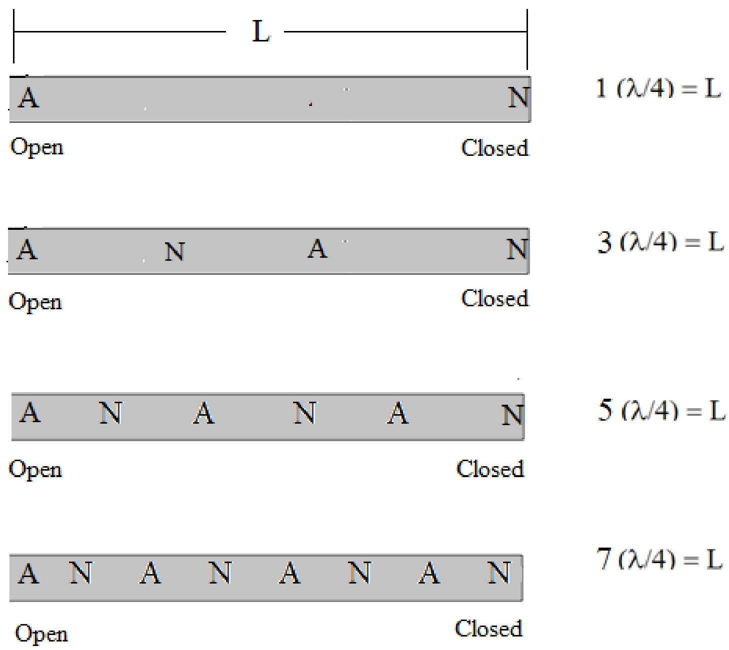
At the closed ends of pipes air cannot vibrate back and force through the end cap: the air there is stationary. These are the locations of the resonating wave's displacement nodes.

Open-Closed Tubes

The trumpet is an example of an open-closed tube; when it is resonating, there is a node at the mouthpiece.	

<u>Example</u>:

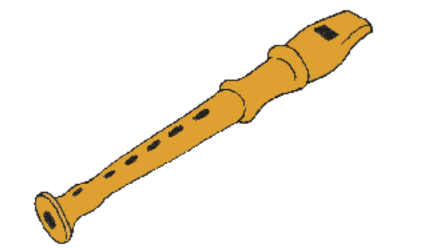
What are the four lowest frequencies of sound that will resonate in a trumpet (an open-closed tube) 0.425 meter long?



$1(\lambda/4) = 0.425$ $\lambda = 1.70$ m $f = 340/\lambda$ $= 100$ Hz	$3(\lambda/4) = 0.425$ $\lambda = (1.70/3)$ m $f = 170/\lambda$ $= 300$ Hz	$5(\lambda/4) = 0.425$ $\lambda = (1.70/5)$ m $f = 170/\lambda$ $= 500$ Hz	$7(\lambda/4) = 0.85$ $\lambda = (3.40/7)$ m $f = 170/\lambda$ $= 700$ Hz

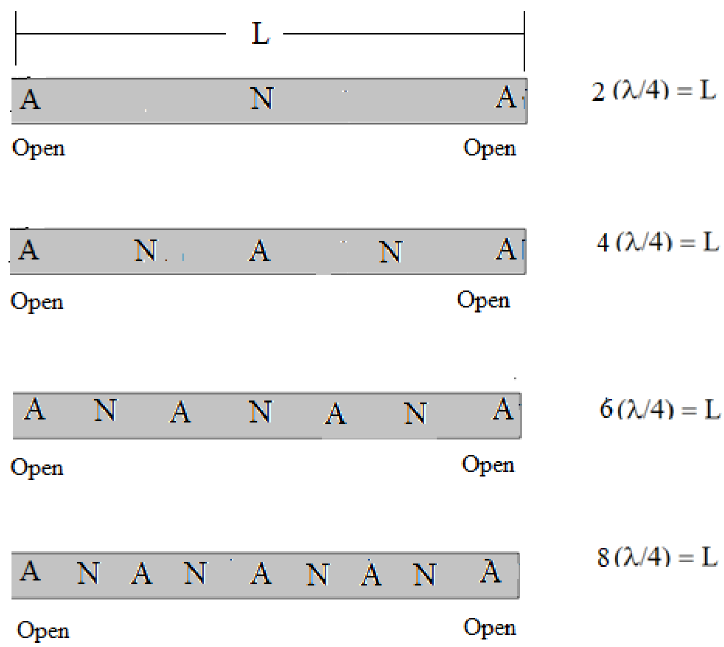
Note: The resonant frequencies are odd-integer multiples of 100 Hz. Other frequencies not shown above include 900, 1100, 1300, 1500, and 1700 Hz, and so on.

Open-Open Tubes

The flute an example of an open-open instrument; when it resonates there is an antinode at both ends.	

<u>Example</u>:

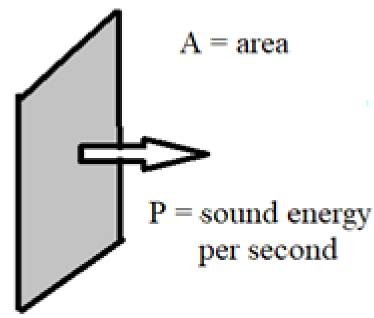
What are the four lowest frequencies of sound that will resonate a 0.17-meter flute (open-open tube)?



$2(\lambda/4) = 0.17$	$4(\lambda/4) = 0.17$	$6(\lambda/4) = 0.17$	$8(\lambda/4) = 0.17$
$\lambda = 0.34$ m	$\lambda = 0.17$ m	$\lambda = (0.68/6)$ m	$\lambda = (0.68/8)$ m
f = 340/0.34 = 1000 Hz	f = 340/0.17 = 2000 Hz	f = 340/0.1133 = 3000 Hz	f = 340/0.085 = 4000 Hz

Note: The resonant frequencies are integer multiples of 200 Hz. Other frequencies include 5000, 6000, 7000, 8000, and 9000 Hz, and so on.

Sound Intensity



I = P/A
Units: W/m^2

<u>Example</u>:

Each second, 4.0×10^{-6} J of sound energy lands on an area A = 2.0×10^{-4} m^2.

What is the sound intensity at that location?

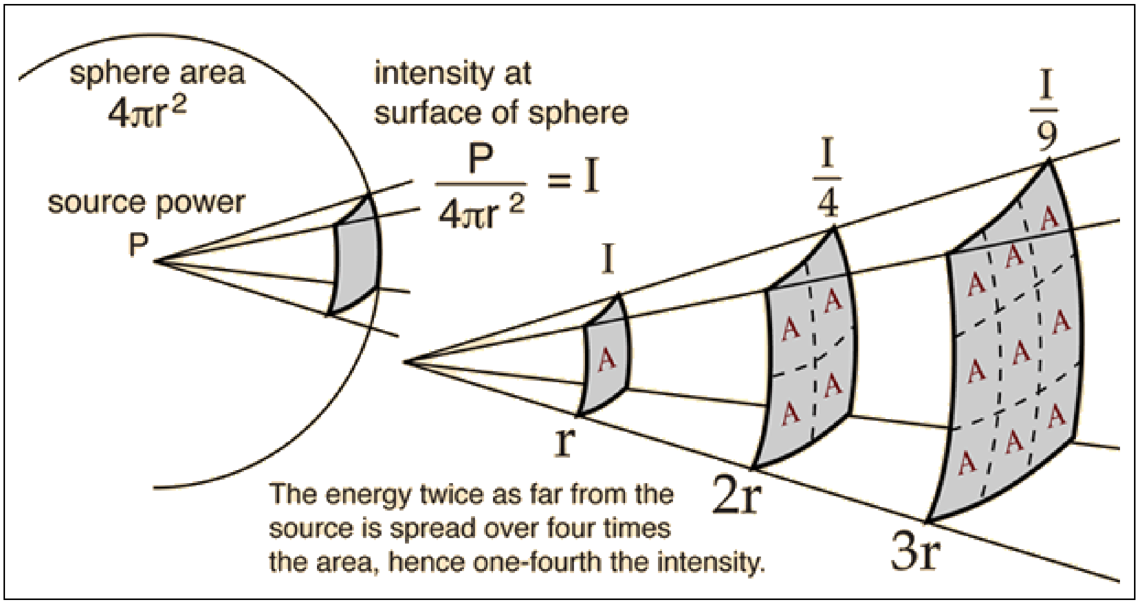
I = P/A
= (4.0×10^{-6} W) /(2.0×10^{-4} m^2)
= 2.0×10^{-2} W/m^2

Spherically-Symmetrical Sound Sources

Sound sources that broadcast equal amounts of energy each second in all directions are called "spherically-symmetric" sound sources.

If the power output of the source of sound is P, and the listener's ear is a distance r away, located on the surface of a sphere radius r and whose surface area is $4\pi r^2$, the ear of the listener receives sound intensity $I = P/4\pi r^2$.

$$I = P/4\pi r^2$$



Quadruple the distance: intensity would be one-sixteenth as much. If the ear is one-tenth as far from the source, the intensity would be 100 times as great.

<u>Example</u>:

A spherically-symmetric speaker has a power output of 200 watts.

What is the intensity 4.0 meters away?

$4\pi(4.0)^2 = 201.06\ m^2$

$I = P/4\pi r^2$
$= 200/201.06$
$= 0.99\ W/m^2$

<u>Example A</u>:	<u>Example B</u>:
How far from a spherically-symmetric 40-watt sound source would the sound intensity be one milli-watt per square meter? 1.0 milliwatt = 1×10^{-3} W $40\text{ W}/(4\pi r^2) = 1.0 \times 10^{-3}\text{ W/m}^2$ $r = 56.42$ m	At a certain distance from a spherically-symmetric sound source the intensity is I. What is the intensity 8, 7, 6, 5, 4 times farther away? Express your answer as a multiple of I. Intensity is proportional to the reciprocal of the distance squared: 1/64 1/49 1/36 1/25 1/16

The Threshold of Human Hearing

The least sound intensity the average healthy human ear can detect is called "The Threshold of Human Hearing," and is $I_o = 1.0 \times 10^{-12}\text{ W/m}^2$ Below this intensity, the ear hears nothing, hence the subscript zero: I_o

Example:

1.5 meters away from a whisper the sound intensity is 2.0×10^{-11} W/m^2.

(a) Treating the whisperer as a spherically-symmetric sound source, what is its output power P?

$4\pi(1.5 \text{ m})^2 = 28.27 \text{ m}^2$

$2.0 \times 10^{-11} = P / 28.27$
$P = 5.65 \times 10^{-10}$ W

This is the approximate output sound power generated by a flapping butterfly wing.

(b) How far from the whisperer would the whisper be inaudible, i.e., $I = I_o$?

$$I = P/(4\pi r^2)$$
$$1.0 \times 10^{-12} = 5.65 \times 10^{-10}/ (4\pi r^2)$$
$$r = 6.71 \text{ m}$$

Decibel Level

One of the two ways we quantify the amount of sound at a point--sound intensity--was discussed previously. The second way of quantifying the amount of sound is to report the "decibel level."

The decibel level, also called "sound level," is calculated as shown below.

$$\beta = 10 \log (I/I_o)$$

	Intensity (W/m^2)	Decibel Level (dB)
Butterfly Wings Fluttering	1×10^{-12}	0
Normal Conversation	3.2×10^{-6}	65
Rock Concert Front Row	0.01	100
Leaf Blower at Waist	0.32	115
AR-15 Rifle Near Ear	10	130

Decibel levels above about 110 dB will impair hearing and perhaps permanently damage the eardrum.

Example:

What is the sound intensity at a point where the decibel level is 36 dB?

If you understand the concept of inverse functions and can apply it to the logarithm function, you would be able to obtain the answer to this question following the steps below. You will get the same answer as other students will who use an equation solver on their calculator. I recommend the solver method.

$$36 = 10 \log (I/I_o)$$
$$10^{3.6} = I/I_o$$
$$I = 3.98 \times 10^{-9}\ W/m^2$$